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TECHNICAL REPORT
WSRL-TR-25/91

BETTERY-TARGET ALLOCATION SUBJECT TO FIRE PLANNING CONSTRAINTS

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SUMMARY(U)

A system is described for allocating artillery batteries to targets for an artillery *Fire Plan*. An optimal allocation, which assigns batteries to targets in the most effective manner, is initially made. This is then modified to ensure that all batteries are firing over the whole duration of the fire plan while preserving the desired effect on each target. The resulting fire plan also satisfies ammunition constraints, ie, a battery is not scheduled to fire more ammunition than what it has. User interaction is supported to allow any specific requirements to be incorporated.

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TABLE OF CONTENTS

LIST OF FIGURES

1. INTRODUCTION

During the course of land warfare, targets are identified which represent threats of various kinds, and which may influence the outcome of a particular battle. Prior to the commencement of an operation, the *force commander* endeavours to locate all enemy positions such as hostile batteries and concentrations of enemy forces which could have an effect on his mission. A list of resources available is next drawn up. The force commander develops a concept of operation and decides on objectives and the type and quantity of resources to be allocated to subordinate commands. This problem of allocating resources by a superior to a subordinate has been discussed by various authors. Witus(ref.1) discusses the problem with reference to the context pertinent to our analysis.

Once the subordinate commander has been assigned the objectives, by which we mean targets, and the resources, which are essentially batteries, the problem becomes that of creating a fire plan. A fire plan is an allocation of batteries to targets, satisfying various criteria. This allocation is subject to many competing factors, and requires considerable effort. Slagle and Hamburger(ref.2), for example, discuss a system called *Battle* in which they consider various factors, pertaining to the battery target allocation, in creating an allocation tree with the aim of making an optimal allocation. This report examines the processes at the level of the subordinate commander. First the effectiveness of each battery against each target is evaluated and then the methods of linear programming are used to obtain an optimal allocation. However the main thrust of this report lies in taking the process a step further, to ensure that all the batteries are firing over the entire fire plan time interval.

2. FIRE PLANNING PROCESSES

There are many types of fire plan, including *Deliberate, Quick, Defensive, Offensive and Counter Battery* types. While the basic form of these fire plans is the same, the specific purpose defines how much detail is included, how much time is available for its preparation, and the type of targets to be engaged.

The allocation process involves selecting a battery to undertake each fire mission, to achieve the desired effect for each target, subject to constraints such as ammunition availability at each fire unit, battery target range, and calibre of weapon for effective engagement, etc.

Deliberate fire plans prepared at the lower tactical levels, such as at company and battalion, are often merged together by the staff at brigade or division headquarters, to form a combined plan for the formation. This merging process is not discussed in this report. However, *Quick* fire plans generally use resources directly available to the planner.

Once a fire plan has commenced, provision must be made to make modifications to the plan to accommodate changes in the tactical situation. For example, batteries may be damaged by counter-battery fire, new targets may emerge, or planned fire may prove to be inadequate. Changes to the plan must be tightly controlled to limit their extent, so that disruption to already distributed orders can be minimised. Plan modification is an essential process in fire planning and should be an integral part of any computer program to assist fire planning. We have not explored the full implications of plan modification. This is left for future development.

3. FIRE PLAN PROFORMA

The basic distribution medium for a fire plan is the fire plan proforma, which lists;

(a) the targets included in the plan, figure 1(a), and

(b) for each fire unit, the fire missions for each target to be engaged, figure 1(b).

The fire missions are laid out on a graphical time schedule showing the relationships between fire missions, the target numbers and the rates of fire for each target, as in figure 1(b).

This layout is useful for informing the fire units of their tasks. However, until fire missions have been allocated to targets, this chart cannot be drawn. The chart can therefore give the planner no assistance in preparing the plan.

An alternate layout which could be useful in visualising the plan requirements prior to battery-target allocation is shown in figure 2. This time chart shows, for each target, the start and stop times for each mission, and can be sorted in ascending start time order.

Batteries can then be allocated to the missions, bearing in mind that a battery can engage only one target at a time, and that a suitable delay must be allowed for when re-laying batteries from one target to another.

The algorithm described in this report uses this alternate layout concept as its starting point. We shall call this the Target Schedule as opposed to the currently used Battery Schedule.

4. BATTERY-TARGET ALLOCATION ALGORITHM

Assuming that batteries have been allocated to each fire mission, (see Section 5 for details), a battery schedule can now be displayed as in figure 3. This allocation is complete in that the weight of fire on the target meets the requirements.

The allocation has a major weaknesses, this being the number of periods when batteries would be firing by themselves, or when few other batteries would be firing at the same time. This is undesirable, as it makes these batteries more vulnerable to detection, and subsequent counter-battery fire. For this reason, fire plans are generally designed so that, as far as possible, all the batteries in the fire plan are active for the entire duration of the plan.

Consequently, the allocation process requires a second step which aims to share the weight of fire on each target among batteries which would otherwise be idle.

This points to the other difficulty with the initial allocation schedule. If care had been taken to ensure that allocated batteries had sufficient ammunition to complete their missions, then the sharing process would change the situation. In fact, the sharing process could be seen as a means of distributing ammunition usage more evenly over all the batteries.

The proposed algorithm therefore involves three steps.

(a) Perform an initial allocation of batteries to targets as described in Section 5, without checking for ammunition sufficiency.

(b) Share fire missions among the batteries by creating supplementary missions to fill gaps in the schedule, Section 6.

(c) Adjust the sharing allocations to ensure each battery has sufficient ammunition for its missions, Section 7.

5. INITIAL BATTERY-TARGET ALLOCATION

5.1 The effectiveness matrix

When allocating batteries to targets the aim is always to achieve an optimal performance. The term 'optimal performance' can be rather deceptive. Its exact meaning depends upon the criteria against which performance is evaluated. As our analysis proceeds, we develop and clarify these criteria. To start with, let us say that our aim is to make an allocation of batteries that inflicts the maximum amount of destruction on enemy targets. Therefore, given a set of batteries and a set of targets, we need to find a way to quantify the effectiveness of each battery against each of the targets. These effectiveness factors would then indicate the appropriate battery-target combination.

There is, however, another detail to be taken care of. A given target may need to be fired upon at different rates of fire, with different types of ammunition, at different points of time, to be effectively neutralised. To accommodate this fact each target is assigned one or more fire missions.

A fire mission has the following characteristics:

> Each fire mission is effective in a particular interval of time, during which a target is fired upon at a particular rate of fire, with a particular type of ammunition.

In a fire plan, a battery is assigned one or more of these fire missions. More precisely therefore, we should quantify the effectiveness of each of the batteries against each of the fire missions. The effectiveness factor should be a measure of the capability of a battery to satisfactorily complete a fire mission. In our analysis the effectiveness depends upon the following factors:

(a) **Battery Location Factor:** This is an integer, between 0 and 9, measuring the vulnerability of a battery to detection and counter battery fire. A well hidden battery is assigned a high value in the scale [0, 9], whereas a battery exposed to direct enemy fire is assigned a value very near to 0.

(b) **Battery Ammunition Factor:** Given a battery and a fire mission, this factor determines whether the battery has enough ammunition to complete the fire mission. It takes values in the interval [0, 1] and is given as follows:

$$\text{Battery ammunition factor} = \frac{\text{(No of rds per gun in the bty)}}{\text{(No of rds per gun needed to complete the fire mission)}}$$

If the above ratio is greater than one then

$$\text{Battery ammunition factor} = 1.$$

(c) **Fire Mission Priority Factor:** This is an integer, between 0 and 9, measuring the importance of a particular fire mission. A target which is perceived to be of a major threat is assigned fire missions with a high priority rating in the [0, 9] scale. Less important targets can be assigned low priority fire missions.

The effectiveness is now defined as the integer part of the product of the above factors

$$\begin{aligned}\text{Effectiveness} = \text{Rounded } [&\text{(Battery location factor)} \\ &* \text{(Battery Ammunition factor)} \\ &* \text{(Fire Mission Priority factor)}].\end{aligned}$$

This definition can be used to calculate the effectiveness of each of the batteries against each of the fire missions. The set of effectiveness values can be arranged in a matrix form similar to the one shown below:

Battery Number	Fire Mission Number 1	2	3
1	5	10	8
2	7	4	15
3	20	3	14

We call this matrix the **effectiveness matrix.**

5.2 The assignment problem

The Problem now is to assign the batteries to the fire missions in a manner which maximises the sum of the effectiveness values associated with each assignment. To illustrate the concept, consider the effectiveness matrix shown above. If now, we assign battery #3 to mission #1, battery #2 to mission #3 and battery #1 to mission #2 then the sum of effectiveness values becomes 20 + 15 + 10 = 45. This would be greater than such a sum associated with any other assignment. The general problem can be stated formally as follows:

> Given an n-by-n array of real numbers {C[i, j]}, where C[i, j] measures the effectiveness of the ith battery against the jth fire mission, find among all permutations {j1,j2,...,jn} of integers {1,2,...,n} that permutation for which the sum C[1, j1] + C[2, j2] + ... + C[n, jn] takes its maximum value.

There are n! possible permutations. The method of choosing the optimum permutation belongs to the theory of linear programming. The solution to the problem can be found in any standard literature on Operations Research; Sasieni, Yaspen and Friedman(ref.3), is a good source.

There is one more detail to be taken care of in the assignment problem. Given n fire missions, one can assign a battery to each fire mission, thereby requiring n batteries to complete the fire plan. This, however, can be wasteful. A battery which has successfully completed a fire mission, would be free to engage any target in a fire mission that starts later. In other words, it is possible to assign the same battery to more than one fire mission, depending upon how the fire missions are distributed over the fire plan time period.

Figure 2 shows a display of 8 fire missions arranged according to their starting time. At no point of time does one require more than 4 batteries to engage all the fire missions active at that point of time. Hence 4 batteries should suffice. Given n fire missions, we first of all find the minimum number m (<= n) of batteries required to complete the fire plan and then assign these m batteries to the n fire missions in a manner that maximises the sum of effectiveness values. A typical assignment is shown in figure 3.

6. FIRE MISSION SHARING

The minimum number of batteries required to complete a given fire plan is always greater than or equal to the number of batteries required at any point of time. This implies that there are time intervals within the fire plan time period, during which one or more batteries are not engaged in any fire mission, see figure 3. This, as said before, is not desirable.

Let us go back to the term 'optimal performance' as mentioned in the previous section. In that section, we fixed criteria against which the performance is to be evaluated. We stipulated that assignment of batteries to fire missions should be such as to maximise the sum of effectiveness values. As this condition does not take into account the fact that there are time periods during which one or more batteries remain idle, we broaden the set of criteria to include the additional stipulation:

The assignment of batteries to the fire missions should be such as to involve all the batteries firing, as much as possible, over the entire fire plan time period.

Of course, the two criteria cannot be satisfied simultaneously. What we propose to do is to make the allocation that maximises the effectiveness-sum, and then to modify this allocation to satisfy the new criterion. To keep all the batteries engaged over the fire plan time period would, in general, require more fire missions than those initially specified. We obtain these additional ones by subdividing the existing fire missions. This results in a new set of fire missions such that, the total effect of this new set on the enemy targets is equivalent to the total effect of the old set of fire missions. Note that this would not be true if we were to create additional fire missions arbitrarily. The following paragraph describes the essential principle behind these subdivisions.

After the optimal assignment as described in Section 5.2 is made, we examine each battery and determine the time intervals during which they are free. Suppose battery B1 is free in the time interval [t1, t2]. We then find all other batteries which are engaged during this period and choose one of them, say, B2. A new fire mission for B1 is created in which B1 fires, at rate one, upon the same target as B2 is firing upon during the interval [t1, t2]. The original fire mission for B2 is altered; the rate of fire is reduced by one. In effect, the original fire mission for B2 is shared with B1 such that the target concerned receives the same amount of fire at the same rate as originally specified. We would refer to such situations by writing that the battery B1 supports the battery B2 in the interval [t1, t2]. We repeat this process for all batteries which have free time, until all the batteries are more or less engaged for the entire fire plan time span. Care is taken to ensure the following:

(a) The number of new fire missions created are kept to a minimum.

(b) The number of times a battery switches from one target to a different one is kept to a minimum.

(c) A time interval of one minute is allowed, to give the gun crew time to realign the guns to a new target. There is nothing rigid about the duration of this interval. It can be increased if required.

Another important aspect of the process described above is ammunition readjustment. When the original fire mission for B2 is subdivided into two fire missions, B1 ends up firing more rounds due to the additional new fire mission and B2 conserves some ammunition as its rate of fire is now reduced by one. This would be a desirable situation if B2 were falling short of ammunition and B1 had excess of it but undesirable otherwise. One might argue, therefore, that new fire missions be created only for those batteries which have an excess of ammunition. However, in a fire plan utilising a number of batteries engaged in a number of fire missions, the process of ammunition readjustment can become rather involved, precluding any such simple conclusions.

Consider the following simple case as depicted in figure 4. The fire plan consists of one target T1 and two batteries B1 and B2 assigned as shown. Suppose both B1 and B2 have just enough ammunition to complete this assignment. B1 is free during the interval [0, 5] and B2 is firing during this period. We would like B1 to support B2 in this interval. B1 could fire at rate one during this interval and the firing rate of B2 could then be reduced by one. The net result, as shown in figure 5, would be that B1 would fall short of ammunition and B2 would have excess of it. However, we could further support B1 with B2 as shown in figure 6. The ammunition now would be completely balanced and both the batteries would be firing over the entire fire plan time span.

The process of subdividing an existing fire mission and creating a new fire mission for a battery, say B, in a time interval in which B was previously free, will be referred to as spreading of B in that time interval. The simple example therefore illustrates the following point. While spreading the batteries, one need not be concerned as to whether the batteries to be spread have spare ammunition or not. The main concern should be to ensure that all the batteries are firing, as much as possible, during the entire fire plan time span.

In big fire plans, spreading will induce an involved redistribution of fire missions which in turn will induce an involved readjustment of ammunition usage in the batteries engaged in the fire missions. This readjustment can become better or worse. However, the number of batteries with ammunition to spare in any fire plan would hopefully be greater than the number of batteries which lack ammunition. Statistically therefore there is a higher probability of obtaining a better readjustment than a worse one.

7. ACCOUNTING FOR AMMUNITION

In spite of the fact that the entire fire plan, after spreading, is in general a better fire plan with a better readjustment of ammunition usage, there could still remain some batteries which would not be able to complete all the fire missions assigned to them, because of lack of ammunition. There can be various factors leading to such a situation. A battery, for example, could have had very little ammunition to start with. This of course, does not constitute a shortcoming of the fire plan as it stands. Rather, this is a shortcoming associated with the initial data. As the aim is to keep all the batteries engaged for almost the entire fire plan time span, each battery allocated to the fire plan should at least start with enough ammunition to be able to fire at rate one for that time span. These deficiencies are better removed at the outset of a fire plan by ensuring sufficient ammunition per battery.

The process of spreading can also lead to a shortage of ammunition in a battery. Since we do not keep track of ammunition limitation during spreading, one or more batteries could get assigned more fire missions than they could support. In such a situation, one of the following courses of action may be adopted:

(a) One could assign extra ammunition to the batteries with shortage and continue with the fire plan as it stands. This has the advantage of implementing a very efficient fire plan. Remember that assigning more ammunition to batteries does not mean that more ammunition is expended in the fire plan. The amount of ammunition to be expended in any fire plan is fixed once the fire missions for each target are determined. Spreading only redistributes the fire among batteries. When we assign extra ammunition to batteries, all it means is that either the fire plan actually requires the excess of ammunition or that the excess amount allocated to some battery will be left as a surplus in the same battery or in some other battery at the end of the fire plan. This is not an undesirable situation because batteries, after completion of a fire plan, are very likely to be assigned to other fire plans that ensue at a latter point of time.

(b) The other course of action, if option (a) is not selected, is for the program to modify the existing fire plan so that all ammunition deficiencies are removed. We do this in a number of steps. Essentially we first select those batteries which have ammunition deficiency. Next, wherever possible, we support these batteries with other batteries which have excess of ammunition. If this is not enough we take off fire missions from batteries that lack ammunition.

8. PROGRAM DESCRIPTION

A program to demonstrate the described fire planning algorithm has been developed. There are two preliminary steps required before creating a fire plan.

(1) The first step is the creation of a list of targets for which fire plans are sought. Only some of these targets will be used for a particular fire plan. Each target in this list is given an identification number called the target number. A suitable description of the target is also included and an eight digit number gives the location of the target with reference to a preassigned frame of reference. This list is called the Master Target List. If a master target list exists, the user may modify the list by adding or deleting targets or by modifying the existing information pertaining to a target.

(2) A Master Battery List is maintained containing details of the batteries available for fire planning. Batteries in this list are identified by the name of the regiment and the fire unit number they are assigned in the regiment. A suitable description, an eight digit location number and a specification of the battery calibre further describe the batteries in the list. A second level description contains the information regarding the amount and the types of ammunition held by each battery.

The program provides for the creation of a new fire plan, in which case the targets to be used in the plan are selected from the Master Target List, and the batteries to be used are selected from the Master Battery List. In addition, already created fire plans can be re-used and modified to incorporate new requirements where necessary.

To create a fire plan, the first step is to select a number of targets from the Master Target List. For each target in this selected list, the user then creates one or more fire missions (in terms of start time, stop time and rate of fire) as described in Section 5.1. Once the set of fire missions is complete, the program calculates and displays the type and the number of batteries required to carry out the fire plan. This information then guides the user to select a set of batteries from the Master Battery List. The program then automatically assigns these batteries to the fire missions as described below. There is one exception to this though; while creating a fire mission the user may specify a certain battery to be allocated to that fire mission. Such fire missions do not go through the processes described below. They are simply allocated the battery indicated.

Given a set of batteries and a set of fire missions the system runs through the following steps:

(i) **Optimal battery - fire mission allocation:** In this step, batteries are allocated to the fire missions in a manner that optimises (maximises) the sum of the effectiveness values in the effectiveness matrix.

(ii) **Subdividing fire mission and battery spreading:** Here some of the existing fire missions are subdivided to create a new and larger set of fire missions. This is done so that all the batteries are engaged over the entire fire plan time span.

(iii) **Adjusting ammunition:** In this step, the fire missions are again altered to take care of batteries which might have fallen short of ammunition during the process (i) and (ii).

The program now displays the allocation as a Battery Schedule, and the user is allowed to make modifications as described in Section 9.

Note that the optimal allocation described in process (i) is taken as a basis for further development. Processes (ii) and (iii) change the optimal allocation to improve its functional efficiency. The final fire plan does not necessarily optimise the sum of effectiveness values.

9. PLAN MODIFICATION

No fire plan algorithm can incorporate all the different situations that are likely to arise in the battle field. The user may deem it necessary to alter parts of the final allocation to account for some of the unaccounted for situations. We make provision for this by allowing the user to

(a) modify a fire mission,

(b) delete a fire mission,

(c) add a new fire mission,

(d) add a new battery,

(e) delete a battery along with the fire missions assigned to it.

When the user is satisfied with the plan, it can be stored and printed for distribution.

10. CONCLUDING REMARKS

We will conclude this document with a few remarks regarding further improvement of the experimental software presented here. We have not attempted to incorporate all the features that could be required in a fully developed fire planning system, but have incorporated sufficient functionality to demonstrate the computing techniques. In particular the following restrictions, which are peripheral to our main purpose, apply to our demonstration program:

(a) All the fire missions in the fire plan are restricted to use High Explosive (HE) ammunition. This essentially makes it simple to keep track of ammunition deficiencies. The program can be expanded to use different types of ammunition with different types of fuse.

(b) The time span of any fire plan is restricted to be within 20 min. A fire plan with a larger time span, would require a wider screen than a normal PC screen, to be clearly displayed. This restriction could be avoided by improving screen management so that it would be possible to scroll horizontally over a wider time span.

A similar restriction to the one mentioned above restricts a fire plan to contain no more than eight batteries. Again this could be avoided by scrolling the screen vertically.

(c) Finally, the software can be upgraded to a real time package to respond to changing situations. For example, if a target were destroyed early in a fire plan then all subsequent fire missions pertaining to that target could be erased and the batteries that were allocated to those fire missions could be re-allocated.

We believe the technique for allocating batteries to fire missions, as described above, has not been investigated before. The demonstration program can form a useful basis to provide functional specifications for a fully engineered fire planning system. The package has been passed to the School of Artillery for evaluation.

11. ACKNOWLEDGMENT

We take this opportunity to thank LtCol P. Burgess, formerly of the School of Artillery, for helpful suggestions.

DEFINITIONS

Battery ammunition factor

Given a battery and a fire mission, this factor determines whether the battery has enough ammunition to complete the fire mission. It takes values in the interval [0, 1] and is equal to the following ratio:

$$\frac{\text{(No of rds per gun in the bty)}}{\text{(No of rds per gun needed to complete the fire mission)}}$$

If the above ratio is greater than one then

Battery ammunition factor = 1.

Battery location factor

This is an integer, between 0 and 9, measuring the vulnerability of a battery to detection and counter battery fire. A well hidden battery is assigned a high value in the scale [0, 9], whereas a battery exposed to direct enemy fire is assigned a value very near to 0.

Battery schedule

This is a chart depicting the fire missions assigned to each battery in a fire plan, as in figure 1(b). The fire missions are laid out on a graphical time schedule showing the relationship between fire missions, the target numbers and the rates of fire for each fire mission. The number in HE/GUN column, is the number of rounds of HE Shells per gun, that the battery will be left with after completing all the fire missions assigned to it.

Effectiveness

This is an integer equal to the integer part of the product of the factors:

(Battery location factor)
* (Battery Ammunition factor)
* (Fire Mission Priority factor).

Effectiveness matrix

A matrix of effectiveness values. See Section 5.1 for illustration.

Fire mission

A fire mission is an assignment for a battery to fire upon a particular target. Each fire mission is effective in a particular interval of time, during which the target is fired upon at a particular rate of fire, with a particular type of ammunition.

Fire mission priority factor

This is an integer, between 0 and 9, measuring the importance of a particular fire mission. A target which is perceived to be of a major threat is assigned fire missions with a high priority rating in the [0, 9] scale. Less important targets can be assigned low priority fire missions.

Fire plan

A fire plan consists of:

(a) A set of batteries.

(b) A set of fire missions.

(c) A battery schedule showing the allocation of batteries to the fire missions.

Fire plan start time

Fire-plan-start-time = Minimum {Start time of all fire missions in the fire plan}.

Fire plan stop time

Fire-plan-stop-time = Maximum {Stop time of all fire missions in the fire plan}.

Fire plan time span

Fire-plan-time-span is the time interval spanned by Fire-plan-start-time and Fire-plan-stop-time.

Force commander

Commanding officer at divisional headquarters.

HE shell

High explosive shell.

Optimal allocation

An allocation of a battery to a fire mission has an effectiveness value associated with it. Given n batteries and n fire missions, an allocation which assigns a battery to each fire mission, will be called optimal, if the sum of the effectiveness values associated with this allocation is not less than such a sum associated with any other allocation.

Optimal allocation set

The set of fire missions obtained after optimal allocation.

Spread allocation

Consider figure I.3 showing an optimal initial allocation of batteries to targets. Not all batteries are firing for the entire fire plan time span. By spread allocation we mean, whenever possible, creating new fire missions for batteries during the period when they are idle. The existing fire missions are altered so that the total effect of all the fire missions after spread allocation is equal to the total effect of the fire missions that existed before spread allocation. See Section 6.

Spread allocation set

The set of fire missions created for Spread allocation. See Section I.2.

Supported And Supporting fire mission

A supporting fire mission is created during spread allocation. It is supporting because it helps reduce the rate of fire in an existing fire mission. The latter is called a supported fire mission. See Section I.2.1(b).

Target schedule

A chart similar to that shown in figure 2. This shows the fire missions prior to battery allocation. The fire missions are sorted according to ascending start time.

REFERENCES

No.	Author	Title
1	Witus, G.	"Decision Support for Planning and Resource Allocation in Hierarchical Organisations". IEEE Transactions on Systems, Man, and Cybernetics, vol.SMC-16, No. 6, pp. 927-942, November/December 1986
2	Slagle, J.R. and Hamburger, H.	"An Expert System for a Resource Allocation Problem". Communications of the ACM, vol.28, No.9, pp.994-1004, September 1985
3	Sasieni, M. Yaspan, A. and Friedman, L.	"Operations Research Methods and Problems". John Wiley and Sons, New York, 1959

TARGET LIST			
Target No.	Location	Description	Remarks
ZT4001	65849365	HILL	
ZT4002	03783421	COPSE	
ZT4003	86743290	TRACK JUNCTION	
ZT4004	87123435	TRENCHES	

Figure 1(a). Target list

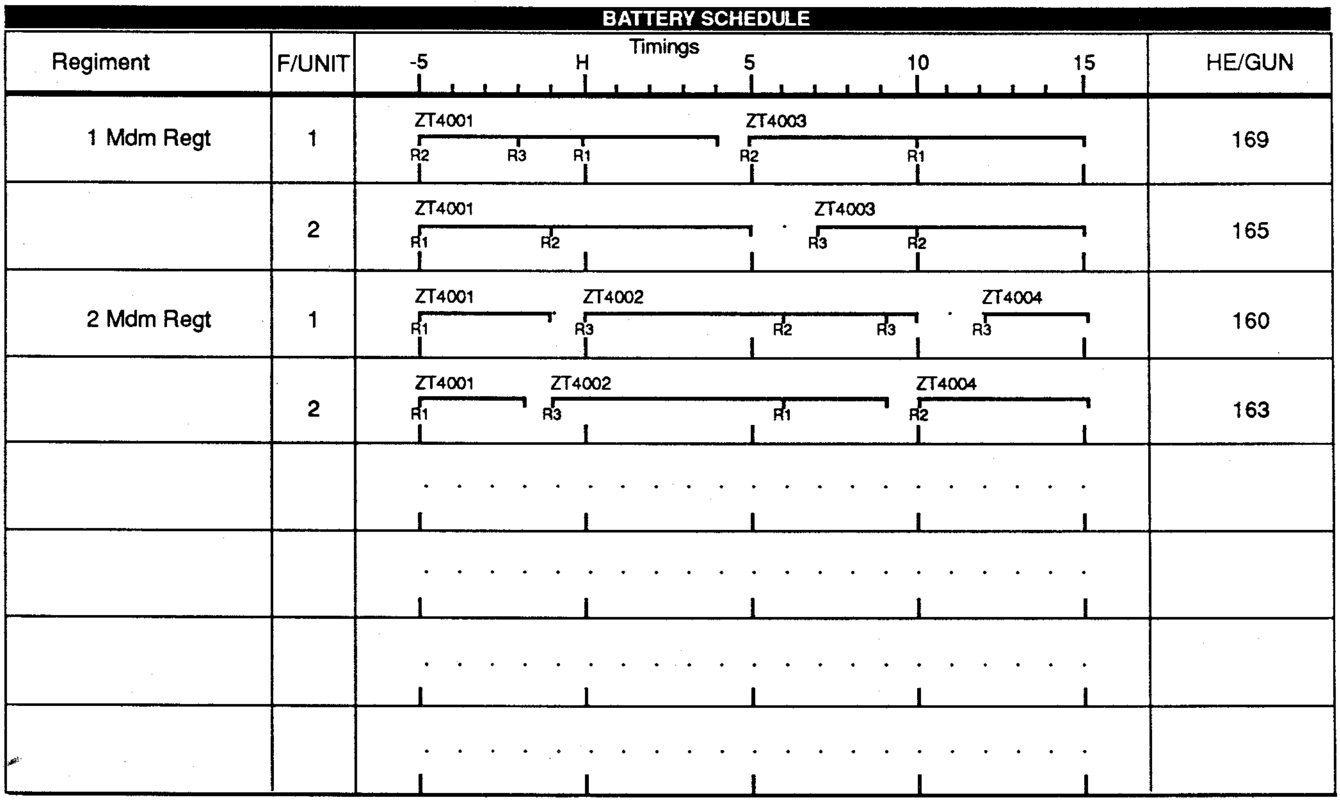

Figure 1(b). Battery schedule

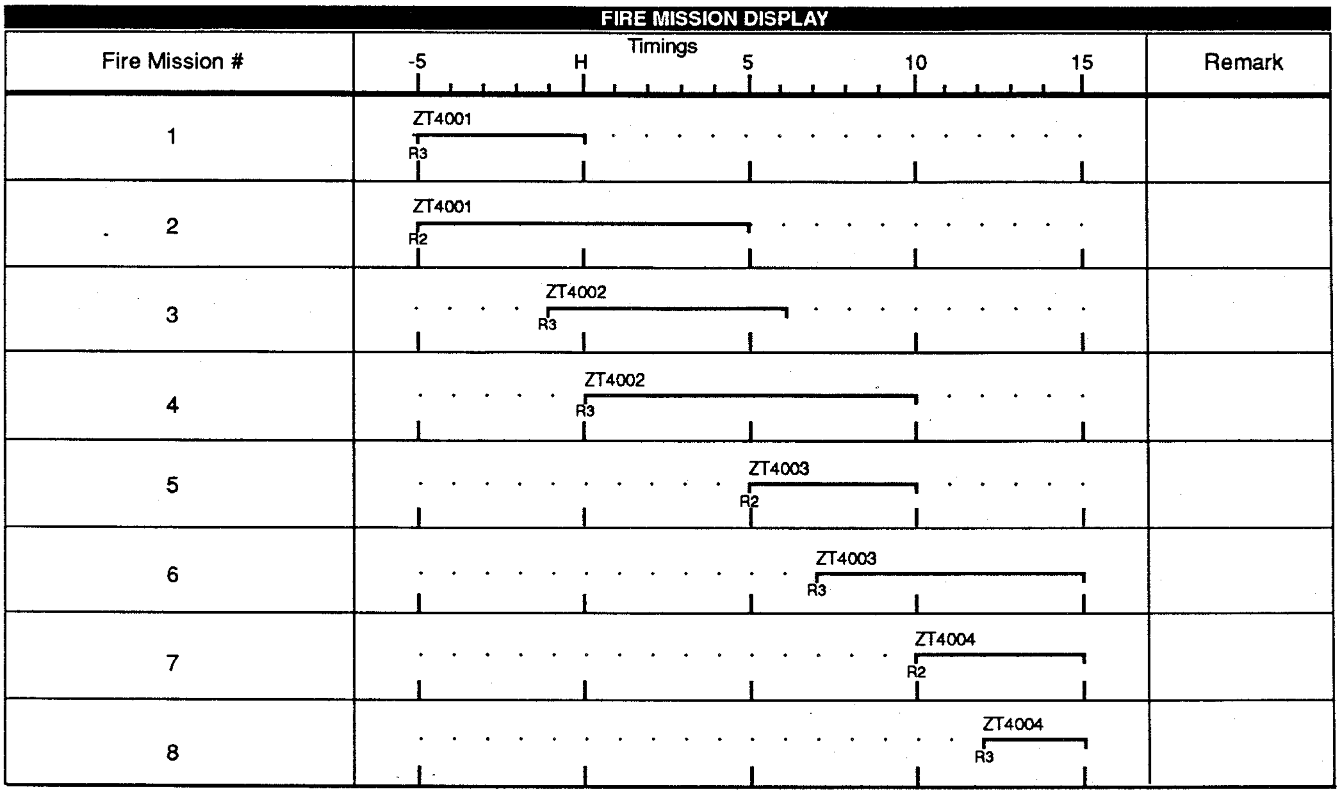

Figure 2. Fire missions sorted according to start time

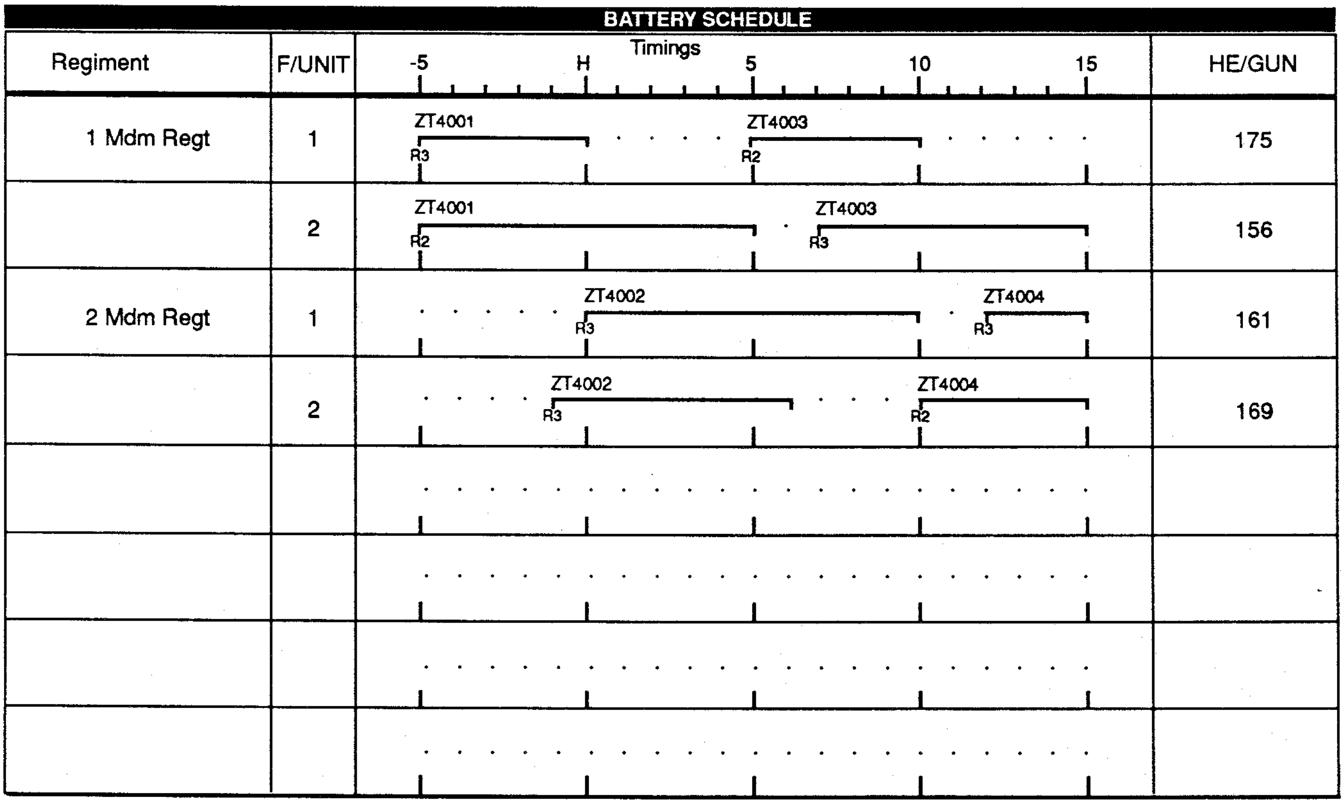

Figure 3. Initial battery schedule showing idle periods

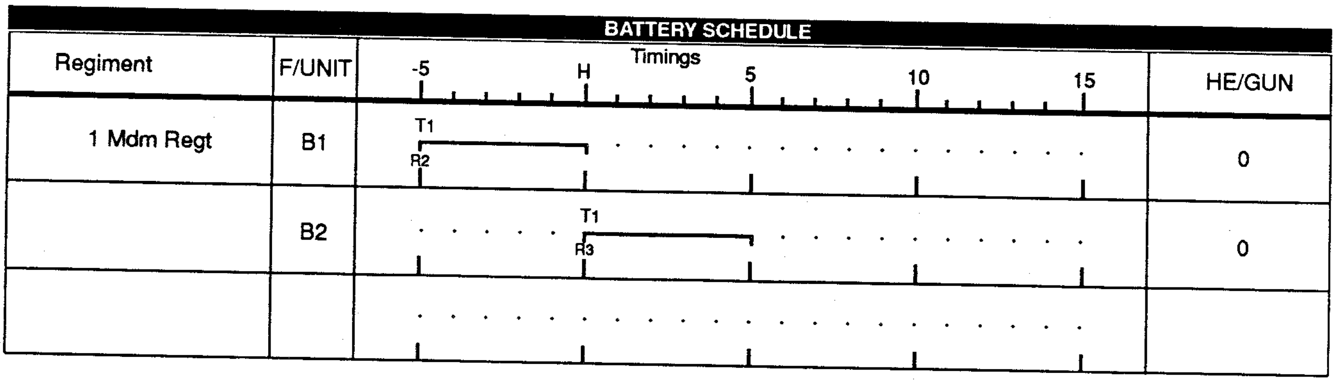

Figure 4. Spreading example - initial battery schedule

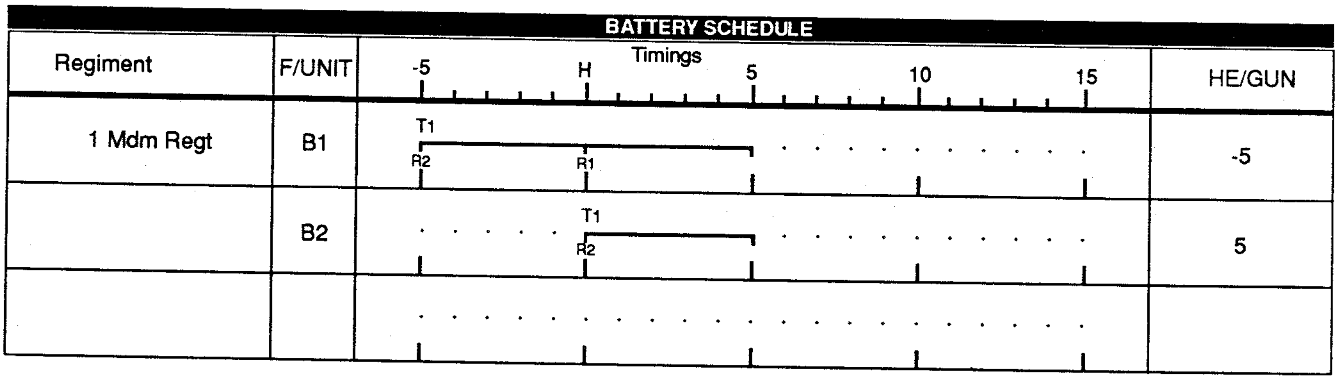

Figure 5. Spreading example - intermediate battery schedule

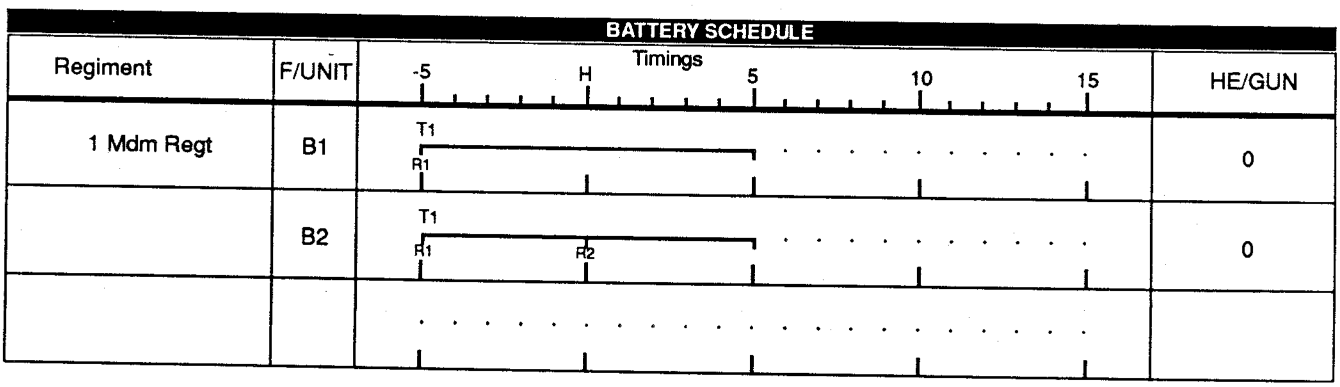

Figure 6. Spreading example - final battery schedule

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APPENDIX I

This Appendix describes the basic logical steps involved in implementing the three steps, (i), (ii), and (iii), mentioned in Section 8.

I.1 Optimal battery - fire mission allocation

The process of assigning batteries to the fire missions proceeds as follows:

I.1.1 Fire missions requiring same calibre guns grouped together

Different fire missions, in general, require different types of guns to achieve their objectives. In this step, fire missions requiring the same kind of guns are grouped together. Let us say, we have a set of fire missions, some of which require 105 mm guns, while the rest require 155 mm guns. We therefore form the following two groups:

105 mm_group: This contains all the fire missions requiring 105 mm guns.

155 mm_group: This contains all the fire missions requiring 155 mm guns.

I.1.2 Minimum number of batteries

Consider the 105 mm_group. Let 'm' be the minimum number of batteries required to engage all the fire missions in this group. Here, we describe the method used to calculate this number.

The **minimum number of batteries** required is that number, which should be able to engage all the fire missions, at any point of time during the fire plan time span and any number less than that should not be able to do so.

Moreover we say that, **a fire mission is active** in a certain time interval if

fire mission start time < end time of the interval and

fire mission stop time + 1 > start time of the interval.

The increased value of the stop time recognises the fact that, a battery, after completing a fire mission, would require at least a minute to be realigned to another target.

To find the minimum number of batteries required, the algorithm first sorts the stop time of the fire missions into a list of ascending order. The earliest start time of all the fire missions is next determined. This number is inserted into the above list of stop times, as its first element. In figure 7, for example, the corresponding list is {-5, 5, 7, 9, 10, 13, 15}. For each time interval spanned by adjacent elements of the above list, the number of fire missions, active in that interval, is determined. The largest number computed represents the minimum number of batteries required.

I.1.3 Form subgroups

We now select m batteries of 105 mm guns from the master battery list. If there were m fire missions in the 105 mm_group then we would just assign the m batteries to the m fire missions in a manner that maximises the sum of effectiveness factor in the effectiveness matrix. However,in general, the total number of fire missions is greater then m. In figure 2, for example, there are 8 fire missions but m is equal to 4. We therefore form subgroups of the 105 mm_group each containing no more then m fire missions. The following describes this process:

> Form the stop time list as mentioned Section I.1.2. Scan the time intervals spanned by adjacent elements of the list and choose that interval, say, [t1, t2], during which the total rate of fire is maximum.
>
> If there are two or more separate subintervals, satisfying the above condition, then choose the one, such that, the total amount of ammunition required to complete all the fire missions that are active during this subinterval, is greater than those required for the other subinterval.
>
> If there is more than one subinterval satisfying the above two conditions, then choose the one, for which the number of active fire missions, is greater than such a number for the other subintervals.
>
> Once the appropriate subinterval is chosen, collect all the fire missions that are active during this subinterval into a subgroup and call this, say, 105 mm_subgroup_1.
>
> After the first subgroup 105 mm_subgroup_1 is created, the process of creating the rest of the subgroups is rather simple. Subtract the fire missions in 105 mm_subgroup_1 from the 105 mm_subgroup. Form the stop time list for the remaining fire missions. The second subgroup, 105 mm_subgroup_2, consists of all the fire missions that are active between the first two elements of the stop time list. The third subgroup is created in a manner similar to that of the second subgroup, after subtracting the first two subgroups from the 105 mm_subgroup. This process continues until all the fire missions are in one or the other subgroup.

The subgroups are stored sequentially as they are formed and batteries are allocated to the subgroups in the same sequence.

> **Remark:**
>
> The motivation behind choosing the subgroups in the above manner is the following. Effectively we are choosing the time interval during which the rate of fire is highest and giving preference to the fire missions active in this interval. This recognises the fact that such an interval is the most important period in the fire plan time span. Most probably, all the targets are engaged during this period. Since it is always more difficult to maintain a higher rate of fire, such a time interval should have the best selection of batteries available at that point of assignment.

I.1.3.1 Illustration of forming subgroups

In figure A1 the fire missions active (see Section I.1.2 for definition) during the interval [5, 7] are #1, #2, #3, #4 and #5. The sum of the rates of fire in these fire missions is equal to 12 and the total ammunition required to complete these fire missions is 125 rounds/gun.

On the other hand the fire missions active in the interval [7, 9] are #2, #3, #4, #5, #6 and #7. Total rates required is again 12 but total ammunition required is 119 rounds/gun. Since all other intervals involve a smaller total rate of fire, we would include the fire missions active during [5, 7] in the first group, as they require more ammunition. If the total ammunition required in the interval [7, 9] were 125 rounds/gun then we would have included these fire missions active in this interval in the first group, because they are more in number than those active during [5, 7].

Next we take off the fire missions included in the first group from the fire plan and are left with fire missions as shown in figure I.2. These would form the second group.

I.1.4 Allocation of batteries

Suppose the 105 mm_group gets divided into three subgroups; first 105 mm_subgroup_1, then 105 mm_subgroup_2, and lastly 105 mm_subgroup_3. We start by collecting all the fire missions in the subgroup 105 mm_subgroup_1 and assign them with batteries from the selected set of m batteries. This assignment is done so that the sum of the effectiveness factors in the effectiveness matrix is maximised.

Our implementation of the algorithm works with a square effectiveness matrix with dimensions not greater than 10-by-10. Each element of the effectiveness matrix is an integer between -999 and 999. Let n be the number of fire missions in a subgroup. Clearly n <= m. The effectiveness matrix in this case has dimensions m-by-m. Let Eff[j, k] denote the effectiveness factor of the jth (1 <= j <= m) battery against the kth (1 <= k <= n) fire mission then

$$\text{Eff}[j, k] = \text{Rounded} \quad \begin{aligned}&(j\text{th battery location factor}\\ &* j\text{th battery ammunition factor}\\ &* k\text{th fire mission priority factor}).\end{aligned}$$

Where the above factors are as defined in Section 5.1.

For (n < k <= m) we define

$$\text{Eff}[j, k] = 0.$$

To explain this fact we note that there are m batteries and n fire missions with n <= m. The algorithm therefore creates (m - n) fictitious fire missions, and assigns the value zero to the effectiveness factors involving these fire missions.

There are a few difficulties to be taken care of in the above definition of the effectiveness value. Allocation of batteries to the first subgroup has the two advantages viz; (i) all the batteries are free for allocation and (ii) all the batteries have nonzero rounds of ammunition. These advantages do not necessarily persist in the allocation to subsequent subgroups.

Let t1 and t2 be the start time and stop time, respectively, of a fire mission, in the subgroup 105 mm-subgroup-2. We say that a battery is not free for this fire mission if the battery is engaged, in any fire mission in 105 mm-subgroup-1, during any subinterval of the interval [t1, t2+1].

If the *j*th battery is not free for the *k*th fire mission then we define

$$\mathrm{Eff}[j, k] \;=\; -999.$$

This is to obviate any possibility of assigning batteries to fire missions, when the batteries are busy elsewhere.

With all the elements of the effectiveness matrix for any particular subgroup now known, the algorithm assigns batteries to the fire missions in the subgroup, by maximising the sum of effectiveness factors. All the m fire missions are now assigned with batteries. However, only the first n fire missions are real. The algorithm therefore stores the assignment of the first n fire missions and discards the rest.

To summarise, the 105 mm-group is subdivided into subgroups. Each of these subgroups is assigned with batteries having 105 mm guns. Next the 155 mm-group goes through the same process and so on. The result is a complete allocation in which every fire mission is assigned a battery. We call this the **optimal allocation** for obvious reason.

Figure I.3 depicts a fictitious optimal allocation. The fire missions are arranged for easy illustration of the rest of the algorithm.

I.2 Spreading the allocation

The set of all fire missions that constitute the optimal allocation will be referred to as the **optimal‾allocation‾set**. In this section, we will alter this set and create an additional set of fire missions to be called the **spread‾allocation‾set.**

Consider figure I.3 showing an optimal initial allocation of batteries to targets. Not all the batteries are firing for the entire fire plan time span. To make the meaning of 'fire-plan-time-span' more precise we make the following definition.

Fire-plan-start-time = Minimum {Start time of all fire missions in the fire plan}.

Fire-plan-stop-time = Maximum {Stop time of all fire missions in the fire plan}.

Fire-plan-time-span now has the obvious meaning.

The algorithm scans each battery and locates every subinterval of the fire-plan-time-span during which the battery is not engaged. Every such subinterval along with the necessary details of the corresponding battery is stored as a record. For example, in figure I.3 the battery named as fire‾unit‾2 in 1 Medium Regiment is free in the interval [-5, 5]. The record stores the battery identifications and the subinterval [-5, 5]. All these records are then stored in a list in a sequential manner such that the record containing subinterval of maximum length occupies the head of the list. Whenever the list is popped, out comes the record with the largest subinterval, at that point of assignment.

Popping the list will give a battery and a time interval during which the battery is free of any fire missions. The algorithm now creates a new fire mission in the free time interval. Section I.2.1 describes this in detail. This process of popping the list and assigning new fire missions, is repeated until either the list runs out of records or a record is reached where the free time interval is less than or equal to 2 min, such time spans being too small for further assignment.

I.2.1 Spreading process

I.2.1.1 Form Categories

Consider a record from the list. Let it contain the battery B1 which is free during the time interval [t1, t2]. Consider all the fire missions in the optimal‾allocation‾set that are active during either the entire interval [t1, t2] or during some subinterval of [t1, t2]. From these, choose those fire missions that require the same type of battery as B1 and which have a rate of fire greater than one. We now divide these fire missions into three different categories.

CATEGORY I: The first category contains all fire missions, whose assigned batteries have a negative amount of ammunition. In other words these batteries do not have sufficient ammunition to complete all the fire missions assigned to them in Section I.1. In figure I.3 fire‾unit 1 in 1 Medium Regiment is such a battery.

CATEGORY II: The battery B1 is free during the time span [t1, t2]. Let T1 and T2 be the targets at which B1 is firing before t1 and after t2 respectively. In the second category we include those fire missions that involve either of the targets T1 or T2.

CATEGORY III: The third category contains all the chosen fire missions. In other words, as a third alternative, we make no distinction among the fire missions.

If the first category is nonempty then the algorithm selects a fire mission from category I. If it is empty, then the algorithm selects a fire mission from the category II. If both the first and the second categories are empty, then the algorithm selects a fire mission from the third category. The selected fire mission is then subdivided to create a new fire mission for the battery B1. If the third category is empty, then the implication is that the original fire plan contains no fire missions, involving batteries of the type B1, in any of the subintervals of [t1, t2]. Hence such batteries cannot be assigned any fire missions in the interval [t1, t2]. They, of necessity, remain idle in that interval.

Category Rationale:

The rationale behind forming the above categories is as follows. When subdividing existing fire missions and creating new ones, the aim is not only to engage battery free time but also to improve the existing fire plan. If a battery in the existing fire plan does not have enough ammunition to complete the fire missions assigned to it and if it is firing during some subinterval of the interval [t1, t2], then support by B1 would take some of the load off the deficient battery. This is the reason behind creating the first category and supporting it first. It must be noted that in keeping with the line of reasoning spelled out in Section 6, we do not check to see if the battery B1 has enough ammunition or not.

If category I is empty, we choose a fire mission from category II. T1 and T2 are the targets, with which B1 is engaged, before and after the interval [t1, t2] respectively. Hence creating a new fire mission for B1, in the interval [t1, t2], which involves either of the targets T1 and T2, will do away with the requirement of re-aligning B1 to a different target either at the beginning or at the end of the spreading fire mission; an obvious advantage.

When both the categories I and II are empty, we just select a fire mission from the set of all relevant fire missions. In other words, there are no preferred groups. This explains putting all the chosen fire missions in category III.

The algorithm now scans the set of fire missions, in the chosen category, to select one according to the rules given below.

I.2.1.2 Selecting fire missions

Recall that the fire missions, in the chosen category, are active in the interval [t1, t2] or in some subinterval of [t1, t2]. The aim is to subdivide these fire missions and create a new fire mission for B1 in [t1, t2].

Apparently, the best selection would be one which provides a fire mission for B1 that spans the entire interval [t1, t2]. Suppose category I is nonempty and we select a fire mission from it, say 'F', consisting of a battery B firing upon a target T. The following question arises - is it possible for B1 to fire upon T for the entire time interval [t1, t2]? The answer to this lies in the following two numbers, s1 and s2, defined as follows

s1	= t1	{if t1 is the start time of the fire plan - or - if the battery B1 is firing upon the target T just before t1}
	= t1 + 1	{otherwise}.
s2	= t2	{if t2 is the stop time of the fire plan - or - if the battery B1 is firing upon the target T just after t2}.
	= t2 - 1	{otherwise}

It is now obvious that B1 will be able to fire upon the target T only in the intervals that are either subintervals of [s1, s2] or identical to [s1, s2]. The factor t1 + 1 in the definition of s1 is due to the fact that the battery B1 would require some time to change over from some target to the target T. Similarly the factor t2−1 in the definition of s2 is due to the time gap required to change over from T to other targets.

To summarise, therefore, we have a battery B1 free in the time interval [t1, t2] and we want to assign it a fire mission, from the chosen category, in that interval. However, once we choose a fire mission involving some target T, the effective free interval of B1 becomes [s1, s2]. The aim is now to select that fire mission which makes the best utilisation of the interval [s1, s2].

The following four steps describe the selection of the appropriate fire mission, from the chosen category.

STEP 1:

The algorithm scans the fire missions in the chosen category, until it finds a fire mission F, in which a battery B is firing upon a target T, at the rate of fire R, in the time span defined by:

start time of F = s1

stop time of F = s2.

That is, the time span of F is exactly equal to [s1, s2]. F is subdivided into two fire missions F1 and F2 defined as below.

(a) **Fire mission F1**: The battery B fires upon the target T, at the rate of fire R - 1, in the time span [s1, s2].

(a) **Fire mission F1:** The battery B fires upon the target T, at the rate of fire R - 1, in the time span [s1, s2].

(b) **Fire mission F2:** The battery B1 fires upon the target T, at the rate of fire 1, in the time span [s1, s2].

Clearly the net effect of the fire missions F1 and F2 on the target T is equivalent to that of the fire mission F. The fire mission F in the optimal¯allocation¯set is replaced by the fire mission F1. The fire mission F2 is inserted into the spread¯allocation¯set.

Supporting fire missions and supported fire missions:

One other consequence of subdividing F into F1 and F2 is that the battery B gets supported by the battery B1. We call F2 the supporting fire mission; battery B1 supports battery B through this fire mission. F1 is called the supported fire mission.

Remark:

Recall that, while grouping the fire missions into categories I, II and III, we chose only those fire missions in which the rate of fire was greater than one. Let us take this opportunity to explain this fact. The fire mission F, in the original fire plan, is being replaced by two new fire missions F1 and F2. However, the fire mission F was originally created, ie B was assigned to T, to maximise the sum of the effectiveness values in the effectiveness matrix - optimal allocation. We do not intend to sacrifice this property altogether while spreading. If the rate of fire in F were equal to one, the new fire mission F1 would be empty. In other words, the assignment of the battery B to the target T would be completely obliterated. With $R > 1$, both F1 and F2 are nonempty and we have

New fire plan = (new optimal¯allocation¯set)

+ (new spread¯allocation¯set).

The new fire plan as a whole, does not retain the property of optimal allocation. Only the first part in RHS still retains that property. The second part is the addition necessary to implement the concept of spreading. This also explains the reason why the rate of fire in the fire mission F2 is equal to one. The sole aim of spreading is to allocate idle batteries, with least disturbance to the original optimal allocation. All fire missions that are created for the purpose of spreading have, therefore, the lowest possible rate of fire. In the rest of the steps that follow, subdivision of fire missions become more involved. However, the basic principles discussed in this remark are still adhered to.

STEP 2:

If there are no fire missions in the chosen category that satisfy the conditions of Step 1, the algorithm rescans the fire missions until it finds a fire mission F in which the battery B is firing upon a target T, at a rate of fire R, in the time span satisfying either of the following conditions:

condition 1

start time of F = s1
stop time of F > s2;

condition 2

start time of F < s1
stop time of F = s2.

That is, the time span of F overlaps either end of [s1, s2].

If condition 1 is satisfied, then the fire mission F in the optimal‾allocation‾set is subdivided into three fire missions F1, F2, and F3 as follows:

(a) **Fire mission F1**: The battery B fires upon the target T, at a rate of fire R - 1, in the time span [s1, s2]. This being the supported fire mission in this subdivision.

(b) **Fire mission F2**: The battery B1 fires upon the target T, at a rate of fire one, in the time span [s1, s2]. This is the supporting fire mission.

(c) **Fire mission F3**: The battery B fires upon the target T, at a rate of fire R, in the time span [s2, stop-time-of-F].

If condition 2 is satisfied then the fire mission F is subdivided into three fire missions F1, F2, and F3 as follows:

(a) **Fire mission F1**: The battery B fires upon the target T, at a rate of fire R, in the time span [start-time-of-F, s1].

(b) **Fire mission F2**: The battery B1 fires upon the target T, at the rate of fire one, in the time span [s1, s2]; the supporting fire mission.

(c) **Fire mission F3**: The battery B fires upon the target T, at the rate of fire R - 1, in the time span [s1, s2]; the supported fire mission.

The three fire missions F1, F2, and F3 have the same net effect on the target T as the original fire mission F. F2 is inserted into the spread‾allocation‾set while in the optimal‾allocation‾set fire mission F is replaced by F1 and F3.

Remark:

In Step 1 the time span of the chosen fire mission exactly coincides with [s1, s2], the effective free time of the battery B1. In Step 2 the time span includes [s1, s2] but it includes it at one end - start time of F = s1 or stop time of F = s2. Hence, as far as spreading of the battery B1 is concerned, both the steps are equally effective; both spread B1 over the entire time span [s1, s2]. The reason behind performing Step 1 before Step 2 is that Step 1 induces less number of changes in the original fire plan than Step 2.

STEP 3:

This step is activated if the algorithm fails to find any fire mission, in the chosen category, that satisfies the conditions of Step 1 and Step 2. Here, the algorithm re-scans the chosen category, until it finds a fire mission F in which the battery B is firing upon a target T, at the rate of fire R, in the time span satisfying the conditions:

start time of F < s1
stop time of F > s2.

That is, the time span of F completely encloses [s1, s2]. F is subdivided into four fire missions F1, F2, F3, and F4 defined as follows.

(a) **Fire mission F1**: The battery B fires upon the target T, at the rate of fire R, in the time span [start time of F, s1].

(b) **Fire mission F2**: The battery B fires upon the target T, at the rate of fire R-1, in the time span [s1, s2]. This is the supported fire mission.

(c) **Fire mission F3**: The battery B1 fires upon the target T, at the rate of fire one, in the time span [s1, s2]; this being the supporting fire mission.

(d) **Fire mission F4**: The battery B fires upon the target T, at the rate of fire R, in the time span [s2, stop time of F].

Again F1, F2, F3, and F4 have the same net effect on the target T as the original fire mission F. F in the optimal¯allocation¯set is replaced by F1, F2 and F4 while F3 is inserted into the spread¯allocation¯set.

Remark:

In this step the time span of the chosen fire mission includes [s1, s2]. However, this inclusion is somewhere in the middle of the time span. This is the reason why the subdivision of F creates more fire missions than the subdivisions in Step 2.

STEP 4:

In the previous steps, the time span of the fire mission chosen was either equivalent to [s1, s2] or contained [s1, s2]. They cover all situations which allow a complete spreading of B1 in the interval [s1, s2]. In the absence of such fire missions in the chosen category, one can only hope for a partial spreading of B1 in the interval [s1, s2]. In this step, the algorithm deals with the case, in which the time spans each of the fire missions in the chosen category, has an intersection with [s1, s2]. Let F be such a fire mission. In F a battery B is firing upon a target T, at a rate of fire R. We define

[u1, u2] = Intersection ([start time of F, stop time of F], [s1, s2]).

The algorithm now scans the chosen category and finds that fire mission F for which the interval of intersection [u1, u2] has the maximum length among all such intervals. The fire mission F is then subdivided to create a new fire mission for B1 in the interval [u1, u2]. This process of subdivision is explained below.

Method of subdivision in Step 4:

The following three cases exhaust all different situations that may arise:

Case 1: Consider the situation which has the following characteristics

s1 <= start time of F
s2 >= stop time of F.

In this case

[u1, u2] = [start time of F, stop time of F].

F is subdivided into two fire missions F1 and F2, defined as follows:

(a) **Fire mission F1:** The battery B fires upon the target T, at the rate of fire R - 1, in the time span [u1, u2]; this is the supported fire mission.

(b) **Fire mission F2:** The battery B1 fires upon the target T, at the rate of fire one, in the time span [u1, u2]. This is the supporting fire mission.

The fire mission F in the optimal allocation set is replaced by F1. F2 is inserted in to the spread‾allocation‾set.

Battery B1 is now partially spread. It is free in the intervals [s1, u1] and [u2, s2]. We store these intervals along with the necessary data of the battery B1, in two records. These records are then stored in the list as discussed at the outset of Section I.2. They are to be retrieved at a latter stage when B1 would get assigned fire missions in the intervals [s1, u1] and [u2, s2].

Case 2: Here consider the situation in which

$$\text{start time of F} < s1$$
$$\text{stop time of F} < s2.$$

Clearly

$$[u1, u2] = [s1, \text{stop time of F}].$$

F is subdivided into fire missions F1, F2, and F3.

(a) **Fire mission F1:** The battery B fires upon the target T, at the rate of fire R, in the time span [start time of F, u1].

(b) **Fire mission F2:** The battery B fires upon the target T, at the rate of fire R - 1, in the time span [u1, u2]; the supported fire mission.

(c) **Fire mission F3:** The battery B1 fires upon the target T, at the rate of fire one, in the time span [u1, u2]. This is the supporting fire mission.

Fire mission F in the optimal¯allocation¯set is replaced by the fire missions F1 and F2 while F3 goes into the spread allocation set.

Battery B1 is still free in the interval [u2, s2]. We store this fact in a record and insert this record in the above mentioned list.

Case 3: Here we consider the situation in which

$$s1 < \text{start time of F}$$
$$s2 < \text{stop time of F}.$$

In this case we have

$$[u1, u2] = [\text{start time of F}, s2].$$

F is divided into three fire missions.

(a) **Fire mission F1:** The battery B fires upon the target T, at the rate of fire R - 1, in the time span [u1, u2]; the supported fire mission.

(b) **Fire mission F2:** The battery B1 fires upon the target T, at the rate of fire one, in the time span [u1, u2]; the supporting fire mission.

(c) **Fire mission F3:** The battery B fires upon the target T at the rate of fire R in the time span [u2, stop time of F].

F1, F2 and F3 taken together are equivalent to F. F1 and F3 replace F in the Optimal¯allocation¯set and F2 goes into the spread¯allocation¯set. Battery B1 is partially spread. It remains free in the interval [s1, u1]. These data are stored in the list for later spreading of B1.

I.2.1.3 An exception

At this stage, we would like to point out an exception to the process of selecting a fire mission, as described in the above section. Suppose category I is empty and category II is nonempty. The algorithm selects a fire mission from category II. Recall that, the rationale behind forming the category II of fire mission, was to do away with the inconvenience of realigning the battery B1 to a different target, either at the beginning or at the end of the new fire mission which spreads B1. The exception we want to make is the following - if the algorithm fails to find a fire mission in category II in either of the steps 1, 2 and 3, it jumps step 4 and selects a fire mission from category III.

The reason behind the above exception is the following. A fire mission that is selected in step 4 has the disadvantage of not providing a complete spreading of the battery B1 in [s1, s2]. By jumping to category III, the algorithm rescans the entire set of fire missions. There is therefore a chance of finding a fire mission which does not satisfy the conditions of category II, but which provides a complete spreading of the battery B1. The exception is justified because a complete spread of B1 is always preferable to the inconvenience of having to realign the battery B1.

I.2.1.4 Dummy spreading

There is one more aspect of allocation to be considered before the algorithm leaves B1. If at the end of what has been done up to now, the battery B1 is still free in the time interval [t1, t2], the algorithm activates a procedure to be referred to as **dummy spreading**. Recall that, to spread the battery B1 in the preceding sections, the algorithm chose those fire missions, that are active in [t1, t2], require the same kind of battery as B1 and have a rate of fire greater than one. In dummy spreading, the algorithm does away with the last condition. In other words, it chooses fire missions, that are active in [t1, t2], require batteries of the type B1 and are firing at rate one. The procedures described in Sections I.2.1.1 and I.2.1.2, are repeated for these fire missions, with one important difference. All fire missions that are created for spreading B1, that is, all supporting fire missions in which B1 supports some other battery, now have their rates of fire equal to zero. Refer to these as **dummy fire missions**. Consequently, the rates of fire in the supported fire missions, are not reduced by one. For example, in step 1 of Section I.2.1.2, the supported fire mission F1 will now have the rate of fire R (= 1) and the supporting fire mission F2, will have a zero rate of fire. The reason for creating the dummy fire missions will become clear, when we discuss the process of ammunition adjustment in Section I.3. These fire missions are removed after ammunition adjustment is complete.

To summarise the algorithm so far, we have allocated batteries to targets in two different stages, satisfying two different principles of allocation. We made the initial allocation in Section I.1, which we call the optimal allocation. In this section we subdivided some of the fire missions in the optimal allocation to create new fire missions for spreading the idle batteries. Let S denote the set of all fire missions that the algorithm creates after all possible spreading. Then S can be written as the disjoint union

$$S = (\text{optimal}^{-}\text{allocation}^{-}\text{set}) + (\text{spread}^{-}\text{allocation}^{-}\text{set})$$

The spread-allocation-set denotes the departure of S from the optimal allocation.

I.2.2 Illustration of spreading

Let us go back to figure I.3. As said earlier, it depicts a fictitious optimal allocation. There are 11 fire missions in the optimal-allocation-set. Here we will illustrate the process of spreading; the fire missions created for spreading will be marked with an "s". For easy reference the batteries will be referred to in an abbreviated form viz fire⁻unit 2 in 1 medium regiment will be referred to as 1-mdm-regt-2. Similarly the first fire mission in the figure, where 1-mdm-regt-1 is firing upon the target ZT4001, at a rate 3, in the interval [-5, 4], will be referred to as {1-mdm-regt-1, R3, [-5, 4]}

Prior to any spreading, the list as mentioned at the outset of Section I.2, contains the following records in the sequence shown:

Record 1: 1-mdm-regt-2 free in [-5, 5]
Record 2: 1-mdm-regt-3 free in [6, 13]
Record 3: 1-fd-regt-2 free in [8, 15]
Record 4: 1-fd-regt-1 free in [-5, 0]
Record 5: 2-fd-regt-2 free in [10, 15]

Other free time intervals are too small to be of any use. They are therefore not stored. For example battery 2-fd-regt-1 is free in [5, 7], this is too small for spreading.

The algorithm starts with the battery 1-mdm-regt-2 free in [-5, 5], and creates the following categories;

Category I contains fire mission {1-mdm-regt-1, R3, [-5, 4]}.
Category II contains fire mission {1-mdm-regt-3, R2, [-5, 6]},
Category III contains both the above fire missions.

The algorithm chooses the fire mission in category I. The effective free time interval [s1, s2] of 1-mdm-regt-2 for this fire mission is [-5, 4]. The situation satisfies conditions in step 1 of Section I.2.1.2. Fire mission P = {1-mdm-regt-1, R3, [-5, 4]} in figure I.3 is subdivided into two fire missions shown in figure I.4 as:

(a) Fire mission P1 = {1-mdm-regt-1, R2, [-5, 4]}; supported fire mission and

(b) Fire mission P2 = (1-mdm-regt-2, R1, [-5, 4]}; supporting fire mission.

The latter is a spreading of 1-mdm-regt-2 and is inserted into the spread-allocation-set. Fire mission P in the optimal-allocation-set is replaced by P1.

Next the algorithm takes up the battery 1-mdm-regt-3 free in [6, 13]. The fire plan now is as shown in figure I.4. Categories I and II both contain the fire mission Q = {1-mdm-regt-2, R3, [5, 15]}. This is therefore chosen for subdivision. The effective free time interval of 1-mdm-regt-3 is [6, 12]. The situation satisfies the conditions of step 3 in Section I.2.1.2. Fire mission Q is subdivided into four fire missions Q1, Q2, Q3, Q4, as shown in figure I.5:

(a) Q1 = {1-mdm-regt-2, R3, [5, 6]}.

(b) Q2 = {1-mdm-regt-2, R2, [6, 12]}; the supported fire mission.

(c) Q3 = {1-mdm-regt-3, R1, [6, 12]}; the supporting fire mission.

(d) Q4 = {1-mdm-regt-2, R3, [12, 15]}.

Q in the optimal-allocation-set is replaced by Q1, Q2 and Q4. Q3 is inserted into the spread-allocation-set.

The fire plan has changed to the one shown in figure I.5. The algorithm takes up the battery 1-fd-regt-2 free in [8, 15]. Categories I and II are empty. Category III contains the fire mission S = {1-fd-regt-1, R2, [10, 15]}. This is chosen for subdivision. The effective free time interval of 1-fd-regt-2 is [9, 15]. Conditions in case 1 of step 4 in Section I.2.1.2 are satisfied. The fire mission S is subdivided into two fire missions S1 and S2 shown in figure I.6 as:

(a) S1 = {1-fd-regt-1, R1, [10, 15]}; the supported fire mission.

(b) S2 = {1-fd-regt-2, R1, [10, 15]}; the supporting fire mission.

Fire mission S, in the optimal-allocation-set is replaced by S1 and S2 is inserted into the spread-allocation-set.

Figure I.6 shows the current fire plan. The algorithm, next considers the battery 1-fd-regt-1 free in [-5, 0]. Category I for this case contains the fire mission V = {2-fd-regt-1, R2, [-5, 5]}. This is chosen for subdivision. The effective free time is [-5, -4]. The situation satisfies the condition 1 of step 2 in Section I.2.1.2. Fire mission V is subdivided into the three fire missions V1, V2 and V3 as shown in figure I.7:

(a) V1 = {2-fd-regt-1, R1, [-5, -4]}; supported fire mission.

(b) V2 = {1-fd-regt-1, R1, [-5, -4]}; supporting fire mission.

(c) V3 = {2-fd-regt-1, R2, [-4, 5]}.

V in the optimal-allocation-set is replaced by V1 and V3. V2 goes into the spread-allocation-set.

Finally, consider the battery 2-fd-regt-2 free in the interval [10, 15], in figure I.7. There are no fire missions in the optimal¯allocation¯set, engaged in this time interval, requiring field regiment (fd-regt) batteries and a rate of fire greater than one. The algorithm activates the procedure of dummy spreading. It creates a dummy fire mission

D = {2-fd-regt-2, R0, [10, 15]}, in support of fire mission

F = {2-fd-regt-1, R1, [7, 15].

The dummy fire mission is drawn as a broken line in figure I.7. However since dummy fire missions have zero rate of fire there is no need to subdivide fire mission F. The optimal¯allocation¯set remains intact and the dummy fire mission D goes into the spread¯allocation¯set.

This completes the spreading of all batteries. Figure I.7 shows the current fire plan. There are 19 fire missions. Five of them marked with s belong to the spread-allocation-set, the rest belong to the optimal-allocation-set.

I.3 Ammunition adjustment

We have been developing the fire plan without paying much attention to the amount of ammunition available with each battery. The task of allocating batteries to targets is now complete. The algorithm next scans each battery and spots those that do not have enough ammunition to complete all the fire missions assigned to them. The particulars of each such battery along with the number of rounds of ammunition by which the guns are falling short are stored in separate records. These records are then stored in a list in a sequential manner such that the record that has the maximum amount of shortage per gun occupies the head of the list. Let us number this as list #0 - the reason for this will become evident very soon. The aim is to pop the list, get the battery with the maximum amount of ammunition deficiency at that point of algorithm and remove this deficiency; repeating the process until all deficiencies are removed.

As discussed earlier, probably the best way of removing ammunition deficiency is by providing extra ammunition; however, if that is not possible, the other way would be to support deficient batteries with batteries having excess of ammunition or as a last resort, take off fire missions from the fire plan. Details of these alternatives are discussed in the following five sections. The first and third sections address the problem of supporting one battery with another and the rest describe the methods of taking off fire missions. The algorithm executes the steps in the order they are given here.

I.3.1 Supporting deficient batteries while preserving optimality

As we have seen in previous considerations, a battery Bi can support another battery Bj if Bi is free in a certain time interval and Bj is engaged in that interval. This kind of support is no more possible. After the optimal allocation, we scanned the fire plan for such opportunities and created new fire missions to spread batteries which were free during some subinterval of the fire plan time span as discussed in Section I.2. These fire missions form the spread¯allocation¯set. If there are still some batteries free in some time interval then the reason might be that, either the time interval is too small, or there are no other batteries firing during that time interval, or the other batteries which are engaged in that time interval are of a different type. These kind of free batteries are of no use as far as support is concerned. The only possibility left is to increase the amount of support in the fire missions contained in the spread¯allocation¯set.

To illustrate the above point, let us for a while go back to step 3 in Section I.2.1.2. In the fire mission F3, the battery B1 is firing upon the target T at the rate of fire one. This is in support to the battery B. Increasing the rate of fire of B1 in F3 will induce a corresponding decrease in the rate of fire of B in F2. This aspect can be used to remove ammunition deficiency in B if B1 has an excess of ammunition. The following steps in the algorithm implement this principle.

Step 1:

To start with, the algorithm pops the record at the head of the list #0. This gives a record containing the particulars of a battery, say, B and the number of rounds by which each gun in the battery falls short. Denote this number by N.

Step 2:

The algorithm then scans the spread¯allocation¯set in search of fire missions that support the battery B. The aim is to increase the rate of fire in these fire missions, which would result in the battery B firing less number of rounds. Only those fire missions in which the corresponding batteries have an excess of ammunition should therefore be chosen - this is in contrast to the principle adopted while spreading the batteries in the last section. Otherwise, while removing ammunition deficiency in the battery B, the supporting batteries will run out of ammunition and add another battery to the problem.

Recall that all supporting fire missions are in the spread-allocation-set, while all fire missions that are being supported are in the optimal-allocation-set; see steps 1, 2, 3, 4, in Section I.2.1.2. Let F1, F2, ..., Fk be the fire missions in the spread¯allocation¯set supporting the battery B. Let B1, B2, ..., Bk be the corresponding batteries in these fire missions having an excess of A1, A2, ..., Ak number of rounds per gun respectively. Let the algorithm scan these fire missions in the order F1, ..., Fk. The next step is to choose one of these fire missions that best serves the purpose of removing ammunition deficiency in the battery B.

Step 3:

Consider the fire mission Fi. Let [ti1, ti2] be the time span during which it is active. Since the associated battery Bi is supporting the battery B, it is evident that B is engaged in the interval [ti1, ti2]. The algorithm scans the optimal¯allocation¯set and finds that fire mission Hi through which B is being supported by Bi in [ti1, ti2]. However B, in the fire mission Hi, might not be firing at a rate of fire that is greater than one in the entire interval [ti1, ti2]. Let [ui1, ui2] be the largest subinterval of [ti1, ti2] in which the rate of fire of B is greater than one.

Remark:

This emphasis on having the battery B firing at a rate of fire that is greater than one is due to the strong desire to preserve optimality in the optimal¯allocation¯set of the fire plan. The aim is to increase the rate of fire of Bi in the fire mission Fi by one and to decrease the rate of fire of B in the fire mission Hi by one. If the battery B is firing at a rate of fire equal to one in fire mission Hi or in parts of Hi then increasing the rate of fire in the supporting fire mission Fi will obliterate Hi or parts of Hi in the optimal¯allocation¯set and replace it by Fi or parts of Fi, thereby destroying the optimality of the set.

Step 4:

To increase the support of Bi to B, the rate of fire of Bi can be increased by one in [ui1, ui2] with a corresponding decrease in the rate of fire of the battery B. Now, recall that Bi has only an excess of Ai number of rounds per gun. Let

$$Ki = \min \{Ai, (ui2 - ui1)\}.$$

Therefore, to increase the support to B, the battery Bi can increase its rate of fire by one in the time span [ui1, ui1 + Ki]. This will result in decreasing the ammunition deficiency of B from N number of rounds per gun to (N - Ki) number of rounds per gun. However, if Ki > N then Bi needs to increase its rate of fire only in the interval [ui1, ui1 + N], as this would suffice to remove the ammunition deficiency completely.

Case 1: The algorithm examines each fire mission in the set {F1, F2, ..., Fk} in the order shown. For each Fi, it finds the corresponding fire mission Hi in which the battery B is being supported by the battery Bi, and calculates the number Ki. This process is continued until a fire mission Fm is reached for which the corresponding number Km >= N. The algorithm alters the fire mission Fm by increasing the rate of fire of Bm by one in the interval [um1, um1 + N] - this is a change in the spread¯allocation¯set. Next the algorithm alters the fire mission Hm by reducing the rate of fire of B by one in the interval [um1, um1 + N] - this is a change in the optimal¯allocation¯set which preserves its optimality. The guns in B now fire N rounds less than they were required to, thus removing the ammunition deficiency in the battery B.

Case 2: If the algorithm fails to find a fire mission Fm for which Km >= N, it scans the entire set {F1, ..., Fk} and finds that fire mission Fs for which the corresponding number Ks = max {K1, ..., Kk}. In other words, it chooses that fire mission which provides the maximum support. Fire mission Fs is altered by increasing the rate of fire of the corresponding battery Bs by one in the time interval [us1, us1 + Ks]. For the battery B, the corresponding fire mission Hs is altered by decreasing the rate of fire of B by one in the interval [us1, us1 + Ks]. The guns in B are now firing Ks rounds less than they were required to. The ammunition deficiency in the battery B is reduced to (N - Ks) rounds per gun. This information, together with the particulars of the battery B, are then stored in a record, which in turn, is stored in the list #0 in proper sequence. As the algorithm examines each member of the list, it will reach the battery B at some point of iteration when, possibly, the rest of the deficiency in ammunition will be removed.

Step 5:

If there are no fire missions of the kind F1, ..., Fk or if there are such fire missions but no intervals of the kind [ui1, ui2], then it is not possible to support the battery B by increasing the rate of fire of some supporting battery in a fire mission in the spread¯allocation¯set. In such cases, the record containing the particulars of the battery B and information regarding its ammunition deficiency is stored in another list numbered as **list #1**.

At the end of step 5, either of the following facts are true - ammunition deficiency in battery B is removed - or ammunition deficiency is partly removed, in which case the appropriate information about B is in list #0 - or ammunition deficiency of B has not changed, in which case the appropriate information about B is in **list #1**.

The algorithm pops list #0 again and repeats the steps 1 to 5 and this process is continued until list #0 is empty. All those batteries whose ammunition deficiency could not be completely removed in this step are now in list #1. List #1 is similar in structure to list #0, the records are similar and are stored in similar sequence.

I.3.2 *Removing fire missions from the spread-allocation-set without reducing fire plan effectiveness*

We will here describe a different process to remove the ammunition deficiencies of the batteries in list #1. Consider a battery in the fire plan. It has a number of fire missions assigned to it. Some of these fire missions are in the optimal¯allocation¯set and the rest are in the spread¯allocation¯set. Any ammunition deficiency of this battery can be removed by taking off some of these fire missions. Taking off fire missions from the optimal¯allocation¯set can destroy its optimality. As we are still attached to that notion, in this section we will explore the possibilities of taking off fire missions from the spread¯allocation¯set. Note that, if a battery has ammunition deficiency then all the fire missions in the spread¯allocation¯set, in which this battery is involved, will have a rate of fire equal to one. This is because such a battery could not have been called upon to render increased support to other batteries in Section I.3.1 and it is only in that section that the rates of fire of fire missions in the spread¯allocation¯set are increased from one to higher values.

There are a few difficulties involved in taking off fire missions from the spread¯allocation¯set. First of all, every fire mission in the spread¯allocation¯set is in support to some fire mission in the optimal¯allocation¯set. For example, if F1 is a fire mission in the spread¯allocation¯set in which a battery B1 is firing at a rate one upon a target T in the interval [t1, t2], then there exists a corresponding supported fire mission H1 in the optimal¯allocation¯set where a battery B2 is firing upon the same target T at a rate R, say, in the interval [t1, t2]. Removing fire mission F1 from the fire plan will result in the target T being fired upon at a rate R in the interval [t1, t2], instead of the required rate R + 1. This can only be offset if the rate of fire in the fire mission H1 is increased by one in the interval [t1, t2]. This is possible if the battery B2 has an excess of ammunition. We would, therefore, remove those fire missions from the spread¯allocation¯set which support fire missions in which the batteries involved have an excess of ammunition.

The other point is that fire missions in the spread¯allocation¯set were originally created to keep the batteries engaged over the entire fire plan time period. Removing some of these will result in some of the batteries remaining idle in portions of the fire plan time span. This cannot be avoided. Batteries that do not have enough ammunition might have to stop firing during some subinterval of the fire plan.

The following two steps describe the removal of ammunition deficiency as envisaged in this section:

Step 1:

To start with, the algorithm pops the record at the head of the list #1. This gives a record containing the particulars of a battery, say B, and the number of rounds by which each gun in the battery falls short - denote this number by N1.

Step 2:

The algorithm then scans the spread¯allocation¯set in search of fire missions that involve the battery B. Let F1, F2, ..., Fn, be the fire missions in the spread¯allocation¯set in each of which B is engaged. Let the algorithm examine these fire missions in that order.

Consider the fire mission F1. Removing this fire mission will enable the battery B to fire, say K1, number of rounds less per gun than it was originally required to, reducing its ammunition deficiency from N1 to N1 - K1 rounds per gun. This is done in the following manner. The algorithm scans the optimal¯allocation¯set to find that fire mission H1 which is being supported by F1. Let T1 be the target which is being fired upon in the fire missions F1 and H1. Let B1 be the battery firing in the fire mission H1 and let B1 have an excess of A1 number of rounds per gun available, after completing all the fire missions assigned to it. Then either of the following is true:

(i) **A1 <= 0:** In this case, the fire mission F1 cannot be removed. The algorithm passes over to the next fire mission F2 with the battery B having the unchanged deficiency of N2 = N1 - 0 rounds per gun.

(ii) **A1 > 0:** In this case, there is an opportunity to reduce ammunition deficiency. Suppose the fire mission F1 has the time span [t11, t12]. Since the rate of fire in F1 is always equal to one, removing F1 will reduce the ammunition deficiency by (t12 ¯ t11) rounds per gun, in the battery B. In order that the target T1, that is being fired upon by both the fire missions F1 and H1, receives the same amount of fire, the rate of fire of B1 has to be increased by one in the interval [t11, t12]. This can be done only if A1 > (t12 - t11). Moreover, since the battery B has a deficiency of N1 rounds, it would suffice to reduce the ammunition deficiency in B by N1 rounds only. Taking all these factors into account, we define the number

$$K1 = \min \{N1, A1, (t12 - t11)\}.$$

The ammunition deficiency in B can therefore be reduced by an amount of K1 rounds per gun.

The algorithm alters the fire mission F1 by reducing the rate of fire of the battery B by one in the interval [t11, t11 + K1]. In other words, the part of the fire mission F1 in the above time span is removed.

Next, the algorithm alters the fire mission H1, by increasing the rate of fire of the battery B1 by one in the time span [t11, t11 + K1].

The algorithm then passes over to consider the fire mission F2 with the battery B having an ammunition deficiency reduced to N2 = N1 - K1 rounds per gun.

Overview

The removal of ammunition deficiency being discussed, in general runs as follows. The algorithm starts with the fire mission F1 and works its way through the set {F1 , ..., Fn}. When it reaches the fire mission Fi, the battery B has an ammunition deficiency of Ni rounds per gun. It finds the fire mission Hi, which is being supported by Fi and calculates the excess number of rounds Ai, of the battery Bi firing in the mission Fi. If Ai <= 0, it passes over the fire mission Fi to the next fire mission Fi+1. If, on the other hand, Ai > 0, it reduces the ammunition deficiency of B by an amount of Ki rounds per gun and passes over to the next fire mission Fi+1, after making appropriate alterations to the fire missions Fi and Hi. This process continues until the algorithm reaches the fire mission Fm, say, where Km = Nm. Here the ammunition deficiency of B is completely removed and the algorithm stops scanning the set {F1, ..., Fn}. If no such fire mission Fm is reached, the algorithm scans the complete set, at the end of which the ammunition deficiency of B is partly removed. From the starting value of N1, the ammunition deficiency of the battery B reduces to the value (N1 - K1 - K2 - ... - Kn). This number together with the particulars of the battery B are stored in another list numbered list #2.

The algorithm again pops list #1 and repeats the above process. This continues until list #1 is empty.

All those batteries whose ammunition deficiency could not be completely removed, are now in list #2. List #2 is similar in structure to the previous such lists.

I.3.3 Supporting deficient batteries with optimality sacrificed

Here we describe a process to manipulate list #2. This is similar to the process described in Section I.3.1 except for one major detail. In that process, we considered a battery B with ammunition deficiency and found a fire mission in the spread‾allocation‾set which supported this battery. We then increased the rate of fire in the supporting fire mission, in some time span [t1, t2], which in turn, eased the ammunition requirement of battery B. There was one catch involved. We had to ensure that battery B fired at a rate of fire greater than one in the interval [t1, t2]. This was to ensure continued optimality of the optimal‾allocation‾set. At the present point in the algorithm, we have used up all possible manoeuvres that can simultaneously remove ammunition deficiency and preserve the optimality of the optimal‾allocation‾set. In this section, we increase the rate of fire of the supporting fire mission in the time span [t1, t2], even when the battery B is firing at rate one in this interval. As a result, a fire mission or a part of it would be removed from the optimal-allocation-set and the optimal‾allocation‾set would cease to be optimal.

At this stage, let us clarify one minor detail. Recall the dummy fire missions in the spread¯allocation¯set, as explained in Section I.2.1.4. These are supporting fire missions having zero rate of fire in support of fire missions in the optimal¯allocation¯set having a rate of fire not greater than one. The reason for this was again to preserve the optimality of the optimal-allocation¯set. We are now willing to sacrifice this aspect of the allocation. In this step therefore, we would increase the rate of fire in the dummy fire missions to support batteries having ammunition deficiency, whenever possible.

Since this section is only slightly different to Section I.3.1 we just point out the differences instead of the details.

(i) To start with, the algorithm pops the record at the head of the list #2. This gives a battery with ammunition deficiency.

(ii) Steps 2 to 5, as described in Section I.3.1, are then implemented, except for the fact that it is not required any more to find that interval [ui1, ui2] where the battery B in the fire mission Hi is firing at a rate of fire greater than one. In fact all occurrence of the interval [ui1, ui2] are now replaced by the interval [ti1, ti2] as defined in that section. Moreover, with that restriction gone, the rate of fire in the dummy fire missions, in the spread-allocation-set, can now be increased to support deficient batteries.

Batteries, whose ammunition deficiency could not be removed in this section are now to be found in list #3. This list is similar to list #1 as described at the end of Section I.3.1. Furthermore list #2 is empty at this point of the algorithm.

I.3.4 Removing fire missions from spread-allocation-set: reduction of fire plan effectiveness

The adjustments to the fire plan in this section and in the one that follows, are rather drastic. We have now exhausted all possibilities by which a battery with ammunition deficiency can be supported with other batteries in the fire plan which have an excess of ammunition. All we can do now, is reduce the number of fire missions or decrease the rate of fire in fire missions in the fire plan to accommodate deficient batteries. The algorithm examines each battery in list #3. The fire missions of these batteries are collected and the rate of fire in each of these fire missions is reduced step by step until all deficient batteries have enough ammunition to complete the respective modified fire missions. The rate of fire in a particular fire mission is reduced by one in each step. Therefore at every step, those fire missions that are firing at rate one, get automatically removed from the fire plan.

Until now, whenever we reduced the rate of fire in a fire mission, say in some time span [t1, t2], we took care to increase the rate of fire in a supporting fire mission in the same time span. The target concerned therefore, received the same amount of fire at the same rate in [t1, t2] after the modification. While reducing the rates of fire in fire missions involving batteries in list #3, it will not be possible to increase support to the concerned targets through other fire missions; we have exhausted all such opportunities in the previous steps. The target concerned will therefore be fired upon at a rate that is less than the rate required by the original fire plan.

Batteries in list #3 would generally have fire missions in both optimal¯allocation¯set and spread¯allocation¯set. In this section, we reduce the rates of fire in fire missions contained in the spread¯allocation¯set. The following steps describe this process:

Step 1:

The algorithm starts by popping the record at the head of the list #3. This gives a record containing the particulars of a battery, say B, and the number of rounds by which each gun in the battery falls short - denote this number by N1.

Step 2:

The algorithm scans the spread¯allocation¯set in search of the fire missions that involve the battery B. Let F1, F2, ..., Fk be the fire missions in the spread¯allocation¯set in each of which the battery B is engaged. Let [ti1, ti2] be the time span of the fire mission Fi during which the battery B is firing at a rate of fire Ri. For the sake of convenience let us write

$$Ki = ti2 - ti1.$$

Let us start with the fire mission F1. Reducing the rate of fire by one in this fire mission will enable the battery B to fire K1 number of rounds less per gun, reducing the ammunition deficiency from N1 to N1 - K1 rounds per gun. However if N1 < K1 we need not do this. The following cases should therefore be considered.

Case 1: If N1 > K1, then the rate of fire in the fire mission F1 is reduced by one. The algorithm passes on to the fire mission F2 with the ammunition deficiency reduced from N1 to N2 (= N1 - K1) rounds per gun. Moreover if R1 = 1, then this process would effectively take the fire mission F1 off the fire plan.

Case 2: If N1 = K1, then again the rate of fire in the fire mission F1 is reduced by one. This completely removes the ammunition deficiency of the battery B. The algorithm passes over to the next record in the list #3.

Case 3: If N1 < K1, then the fire mission F1 is subdivided into two fire missions F11 and F12. Fire mission F11 has the time span [t11, t11 + N1] during which it fires at the rate of fire R1 - 1, while fire mission F12 fires during the time span [t11 + N1, t12] at a rate of fire R1. Owing to the reduced rate of fire in F11, the battery gains N1 rounds per gun. This completely removes its ammunition deficiency. The algorithm passes over to the next record in the list #3.

The general procedure in step 2 is therefore as follows. The algorithm starts with the fire mission F1 and continues through the set {F1, ..., Fk}. When it reaches the fire mission Fi, the battery B has an ammunition deficiency of Ni rounds per gun. The algorithm now reduces the rate of fire of the fire mission Fi by one and passes on to the fire mission Fi + 1 with an ammunition deficiency of Ni + 1 = Ni - Ki. This process continues until the algorithm reaches a fire mission, say Fm, where Nm <= Km. Here the ammunition deficiency of B is completely removed and the algorithm passes on to the next record in list #3.

Step 3:

If no fire missions of the kind Fm can be found, the algorithm works up to the fire mission Fk. The ammunition deficiency of the battery B is only partly removed and some of the fire missions in the set {F1, ..., Fk} are completely removed from the fire mission. The algorithm goes back to the fire mission F1 and starts the process in step 2 all over again. If a fire mission Fi has been removed, the algorithm passes from the fire mission Fi - 1 to the fire mission Fi + 1. This process continues until either the ammunition deficiency of the battery B is completely removed or all the fire missions in the set {F1, ..., Fk} are removed from the fire plan. In the latter case, the situation is as follows:

All the fire missions in the spread¯allocation¯set involving battery B have been removed and B still has ammunition deficiency of, say, M rounds per gun; this number together with the particulars of B are then stored in another list numbered as list #4.

The algorithm repeats the above steps until list #3 is empty.

All those batteries whose ammunition deficiency could not be completely removed in this section are now in list #4. List #4 is identical in structure to previous lists of the same type.

I.3.5 Removing fire mission from optimal-sllocation-set

List #4 contains all those batteries whose ammunition deficiency has survived the previous modifications. These batteries no more have any fire missions in the spread¯allocation¯set. In this step, the algorithm removes the ammunition deficiency of these batteries by removing fire missions or by reducing the rate of fire in the fire missions contained in the optimal¯allocation¯set. The details involved are identical to those described in step¯D except that the algorithm now modifies the optimal¯allocation¯set instead of the spread¯allocation¯set.

The above five steps complete the process of removing ammunition deficiencies from batteries in the fire plan.

I.3.6 Illustration of ammunition adjustment

To continue with the example discussed in Section I.2.2, consider figure I.7 again. Recall that figure I.7 represents the fire plan after complete spreading. The spread-allocation-set contains the fire missions marked with an "s" and the rest of the fire missions belong to the optimal-allocation-set.

There are three batteries which lack ammunition to complete the assigned fire missions. The algorithm stores this information in list #0 in three records in the following sequence:

	Battery	Deficiency
Record 1:	2-fd-regt-1	11 rounds/gun
Record 2:	1-mdm-regt-2	6 rounds/gun
Record 3:	1-fd-regt-2	1 round/gun

Illustration of Section I.3.1:

The algorithm pops the record 1 and scans the spread-allocation-set for a fire mission that supports the battery 2-fd-regt-1. There is one such fire mission viz. fire mission F1 = {1-fd-regt-1, R1, [-5, -1]} in figure I.7. This fire mission supports the fire mission F2 = {2-fd-regt-1, R1, [-5, -1]} in which 2-fd-regt-1 is engaged. However fire mission F2 is firing at rate 1. The ammunition deficiency of 2-fd-regt-1, therefore, cannot be removed through the procedure described in Section I.3.1. Record 1 goes into list #1.

The algorithm then pops record 2 and scans the spread-allocation-set for a fire mission, that supports the battery 1-mdm-regt-2. There is one such fire mission viz. F1 = {1-mdm-regt-3, R1, [6, 12]} in figure I.7.

To explain the notations used in Section I.3.1, we note that fire mission F1 is firing in [t11, t12] = [6, 12]. The corresponding battery, 1-mdm-regt-3 has an excess of A1 = 14 rounds/ gun. F1 supports the fire mission H1 = {1-mdm-regt-2, R2, [6, 12]}. Hence [u11, u12] = [6, 12] and we have

$$K1 = \min(A1, (u12 - u11)) = \min(14, 6) = 6$$

The rate of fire of F1 is increased by one and the rate of fire of H1 is decreased by one. Battery 1-mdm-regt-2 gains 6 rounds/gun in this manoeuvre which removes its ammunition deficiency. The altered fire plan is shown in figure I.8.

The algorithm then pops the record 3. It is easily seen that the ammunition deficiency of the battery 1-fd-regt-2 cannot be removed through procedures described in Section I.3.1. Record 3 is therefore inserted in list #1. As list #0 is now empty, the algorithm passes on to the procedure in Section I.3.2.

Illustration of Section I.3.2:

List #1 contains record 1 and record 3 in this order. Record 1 involves the battery 2-fd-regt-1. As can be seen in figure I.8, there are no fire missions in the spread‾allocation‾set that involve this battery its ammunition deficiency cannot be removed in this step. This record, therefore, goes into the list #2.

Next the algorithm pops the record 3. The battery 1-fd-regt-2 has the fire mission F1 = {1-fd-regt-2, R1, [10, 15]} in the interval [t11, t12] = [10, 15] in the spread‾allocation‾set. The fire mission that is being supported by F1 is the fire mission H1 = {1-fd-regt-1, R1, [10, 15]}.

In the notation of Section I.3.2 we have B1 = 1-fd-regt-1. B1 has an excess A1 = 11 rounds of ammunition per gun. Hence

$$K1 = \min(1, 11, 15\text{-}10) = 1.$$

The algorithm alters fire mission F1 by reducing the rate of fire by one in the interval [10, 10 + 1]. See figure I.9. Part of the fire mission F1 is therefore removed and the ammunition deficiency of the battery 1-fd-regt-2 is adjusted. Next, the rate of fire in the fire mission H1 is increased by one in the interval [10, 11]. As list #1 is now empty the algorithm passes on to the procedure in Section I.3.3.

Illustration of Section I.3.3:

List #2 is now popped to give record 1. The algorithm scans the spread‾allocation‾set for fire missions supporting the battery 2-fd-regt-1. There are two such fire missions, F1 = {1-fd-regt-1, R1, [-5, -1]} and fire mission F2 = {2-fd-regt-2, R0, [10, 15]}; these are active in the time spans [t11, t12] = [-5, -1] and [t21, t22] = [10, 15]. The corresponding batteries in these fire missions have excess ammunition of the amount 10 rounds 15 rounds/gun respectively; A1 = 10 and A2 = 15. We have the fire missions H1 = {2-fd-regt-1, R1, [-5, -1]} and fire mission H2 = {2-fd-regt-1, R1, [7, 15]} in the optimal allocation set in which battery 2-fd-regt-1 is being supported by F1 and F2. H1 and H2 are firing at rate one as it should be for cases in this section.

The rate of fire of the fire mission F2 is increased by one and a part of the fire mission H2 is removed, see figure I.10. The ammunition deficiency of 2-fd-regt-1 is reduced from 11 rounds to 11-5 = 6 rounds/gun.

Next, the rate of fire of F1 is increased by one and the fire mission H1 is removed from the optimal-allocation-set. This reduces the ammunition deficiency of 2-fd-regt-1 from 6 rounds to 2 round/gun as shown in figure I.11.

Record¯1 is altered to:

record¯1	2-fd-regt-1	1 round/gun

Since both the fire missions F1 and F2 are now used, record 1 is inserted into list #3.

Illustration of Section I.3.4:

Since the battery 2-fd-regt-1 is not engaged in any fire missions included in spread¯allocation¯set, no ammunition deficiency is removed in this step. Record 1 is inserted into list #4.

Illustration of Section I.3.5:

Here the ammunition deficiency of 2-fd-regt-1 is completely removed by altering the fire mission, {2-fd-regt-1, R2, [-1, 5]} in figure I.11, to the fire mission {2-fd-regt-1, R2,[0, 5]} shown in figure I.12. The altered fire mission belongs to the optimal-allocation-set. The final allocation is depicted in figure I.12. Ammunition deficiency is now completely removed.

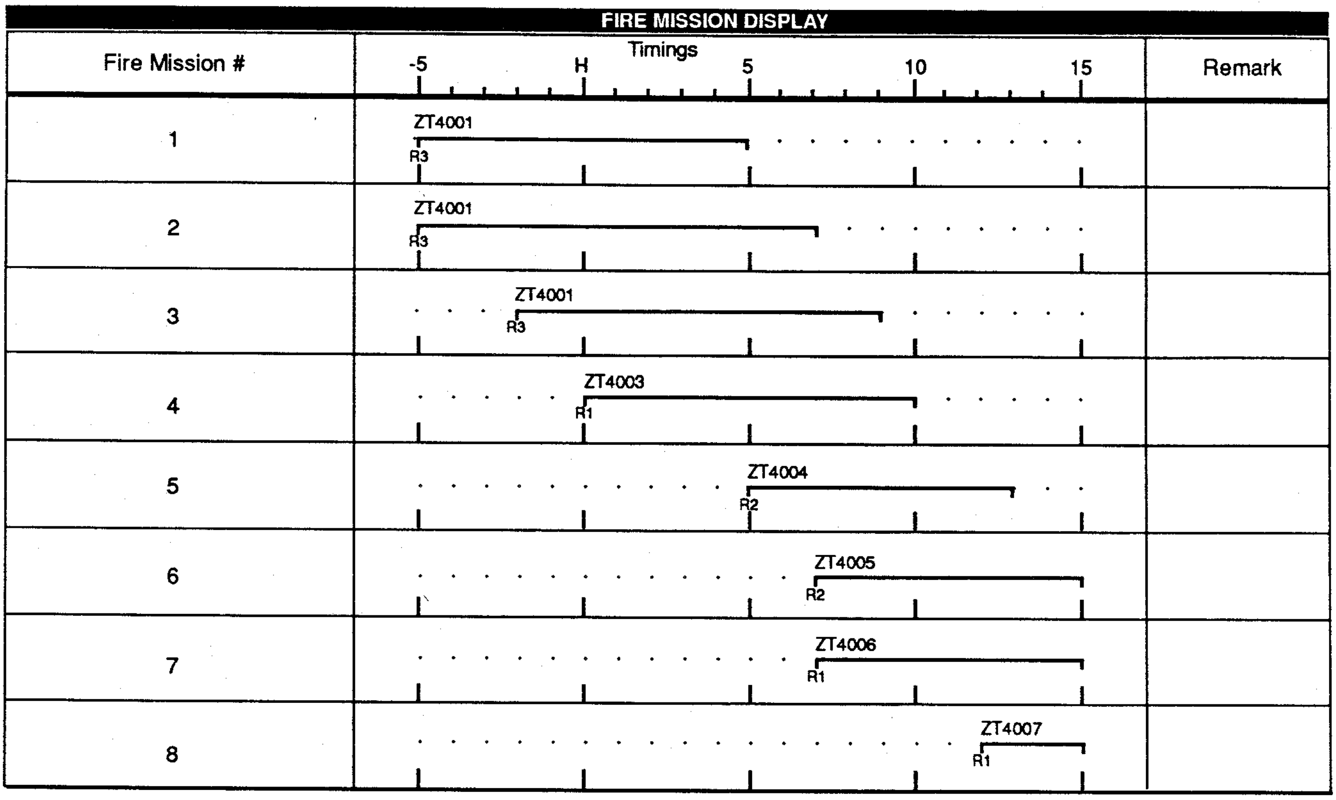

Figure I.1 Forming subgroups: fire mission No 1 to No 5 in first subgroup

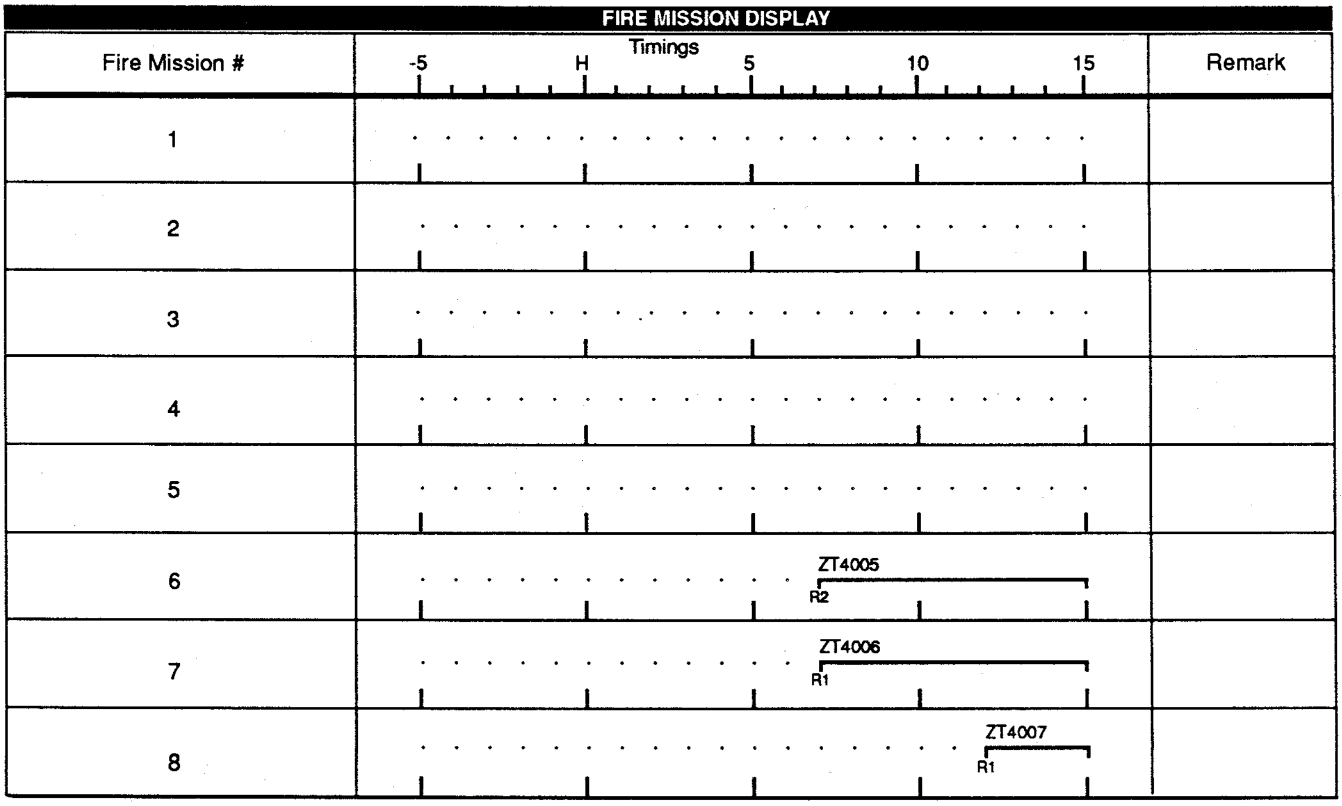

Figure I.2 Forming subgroups: fire missions in second subgroup

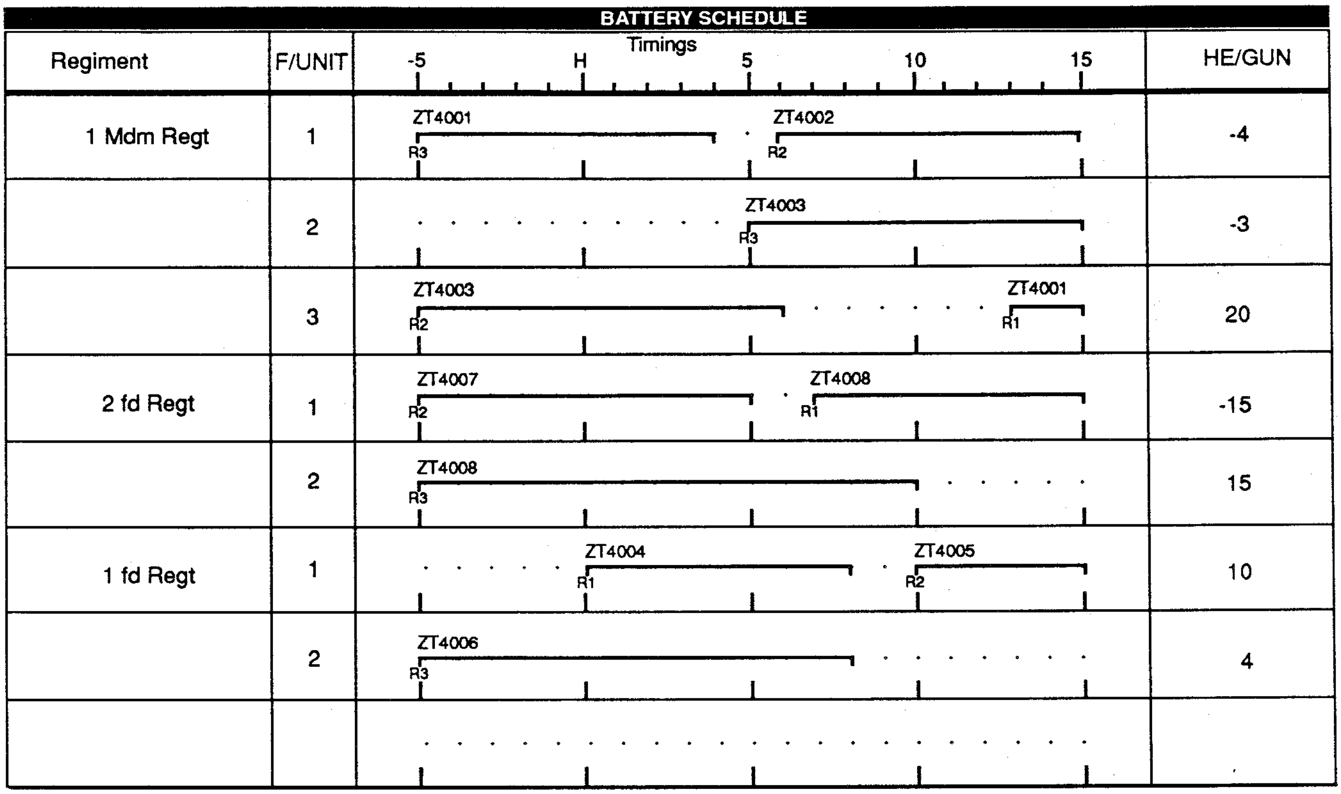

Figure I.3 An optimal allocation

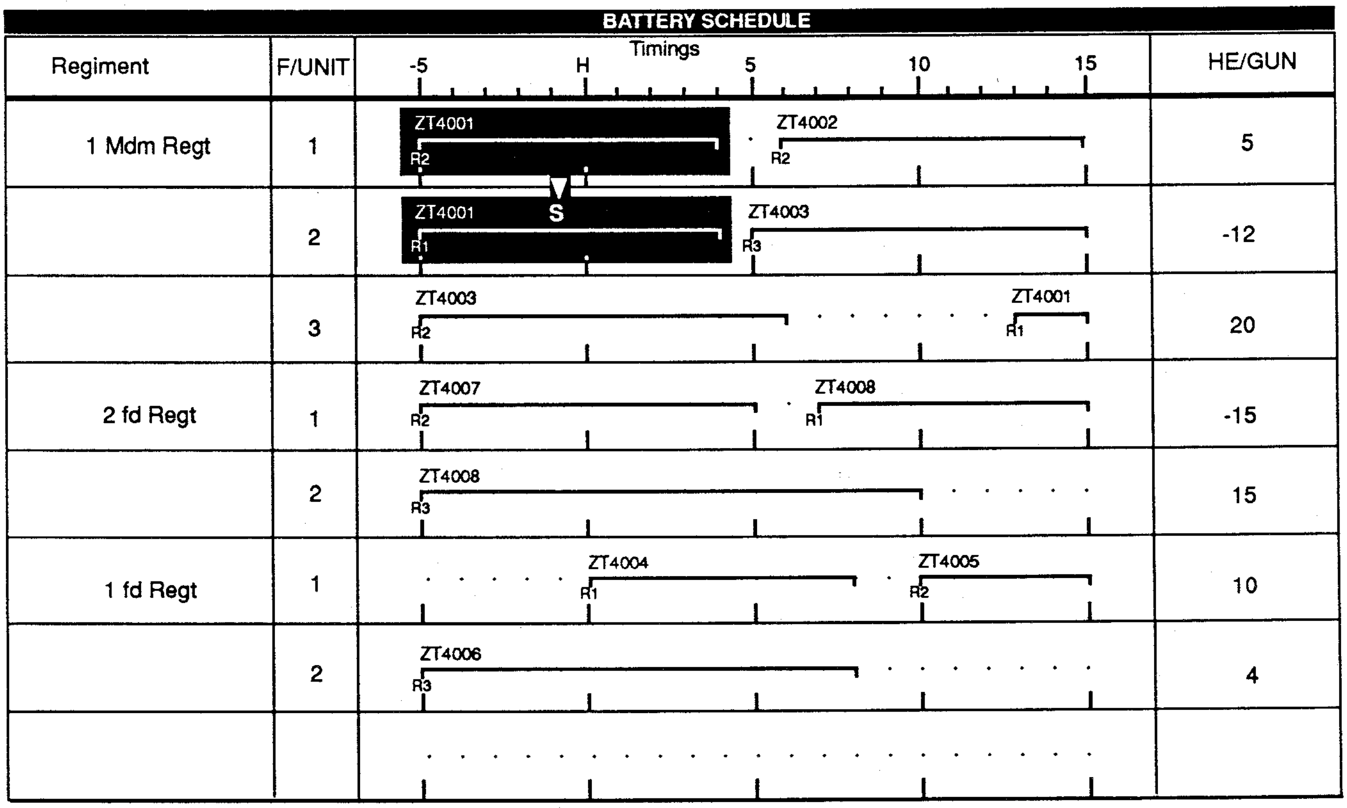

Figure I.4 Spreading stage 1

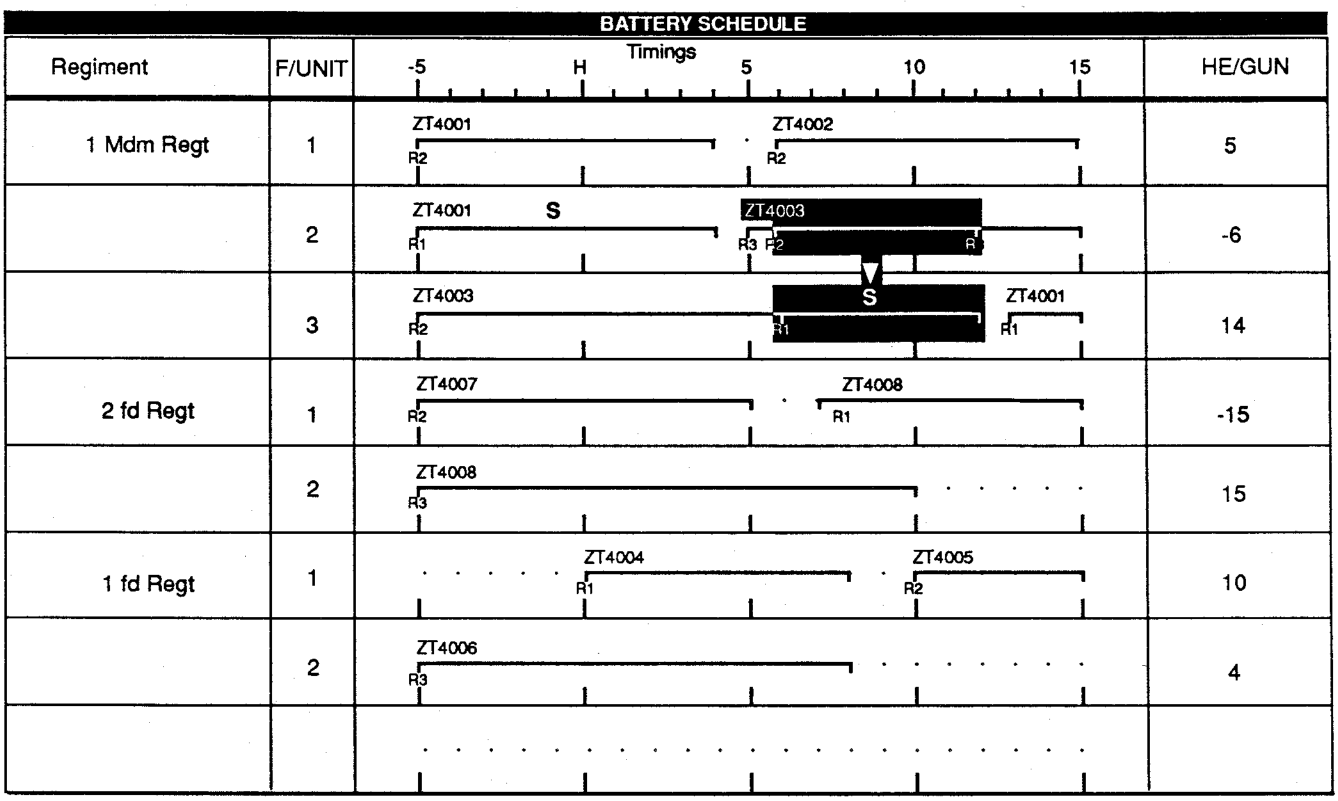

Figure I.5 Spreading stage 2

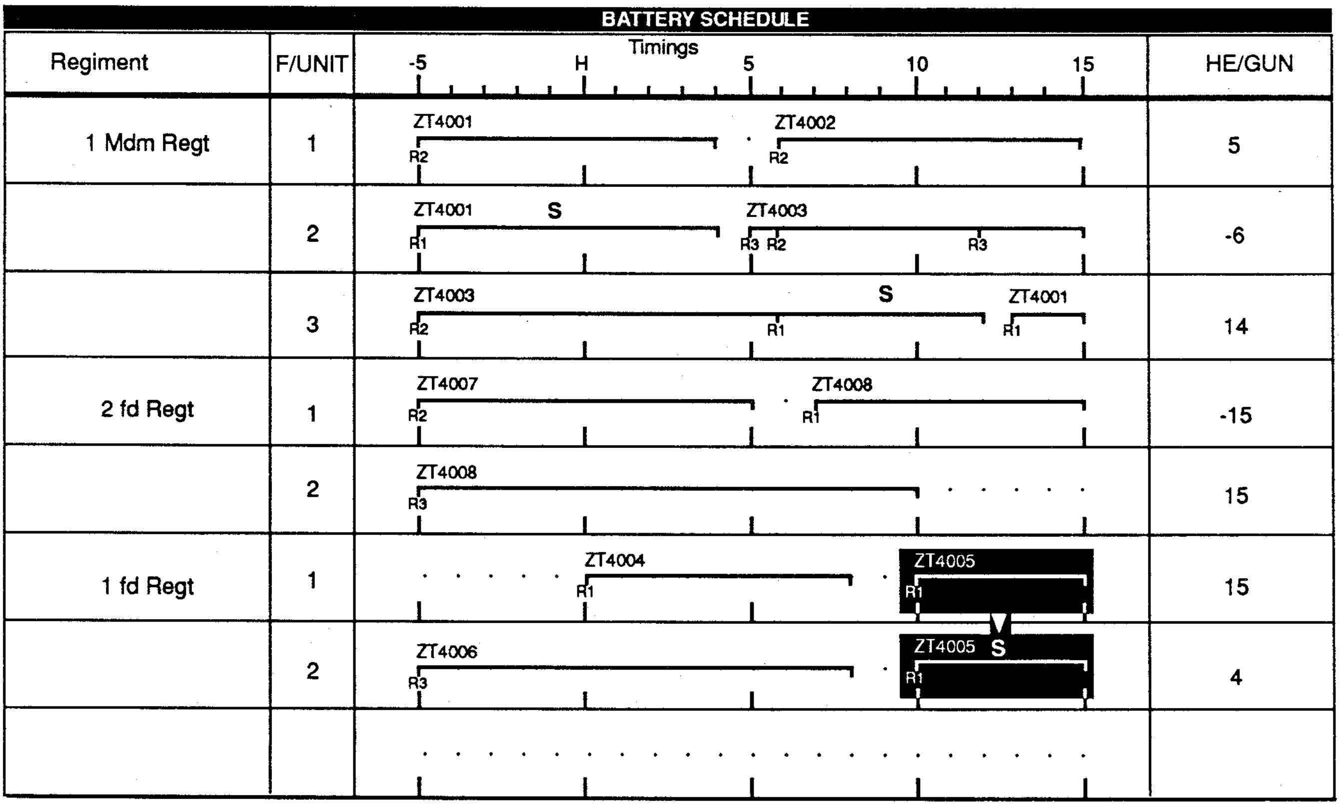

Figure I.6 Spreading stage 3

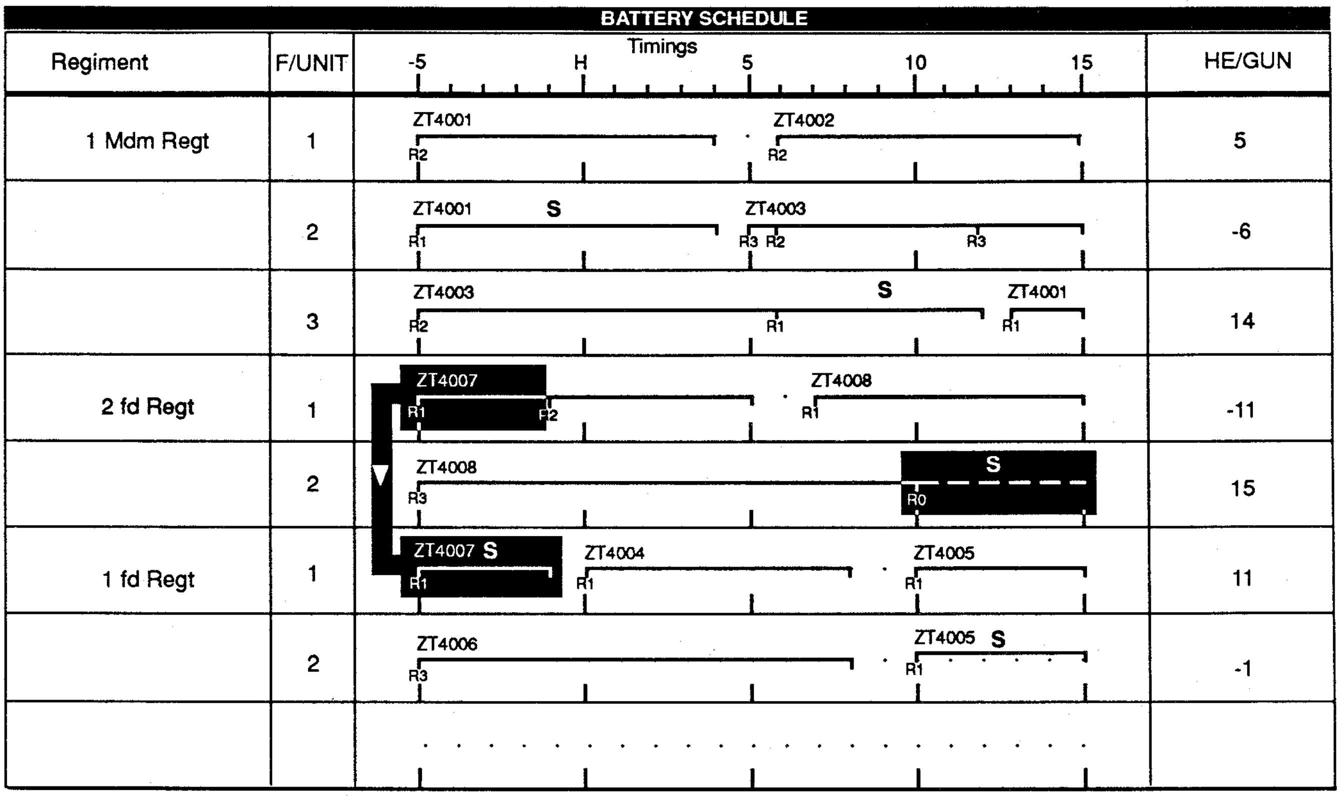

Figure I.7 Spreading stage 4

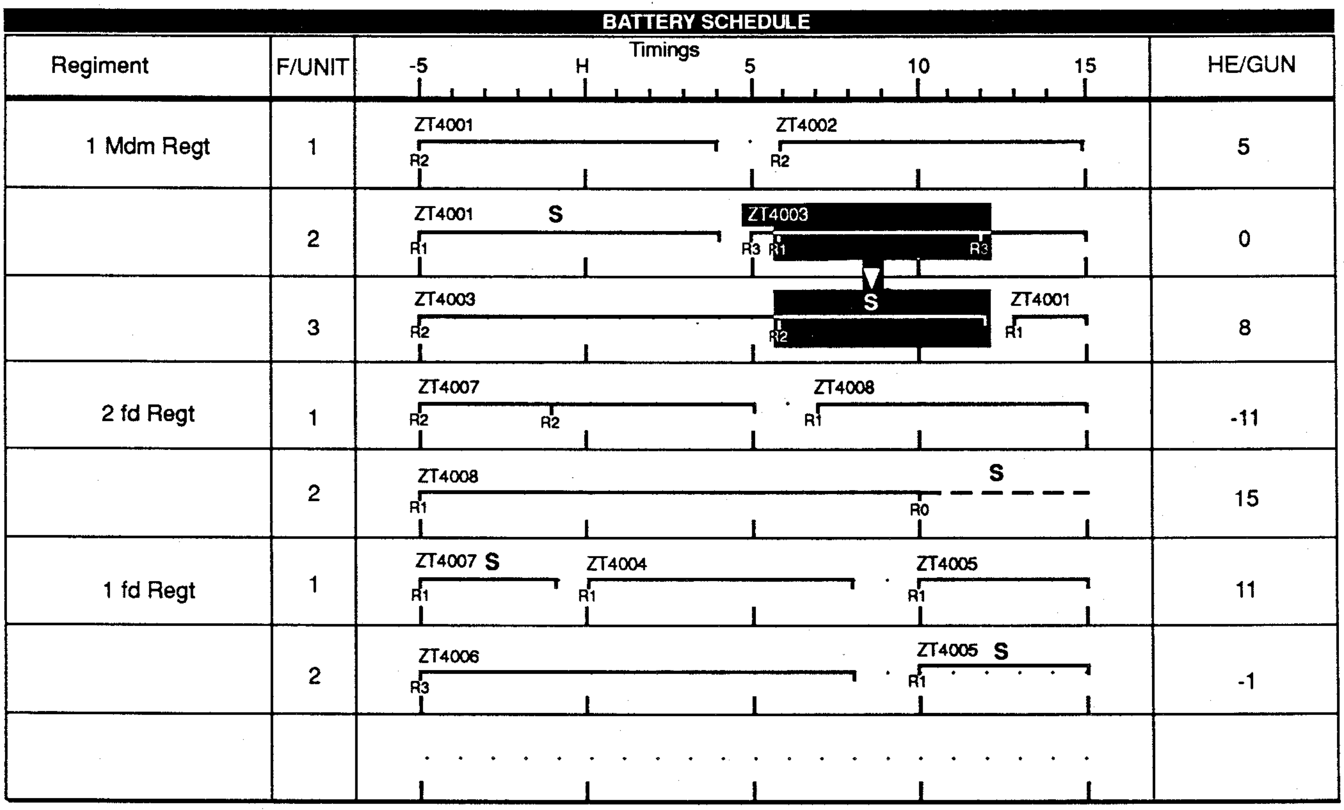

Figure I.8 Ammunition adjustment stage 1

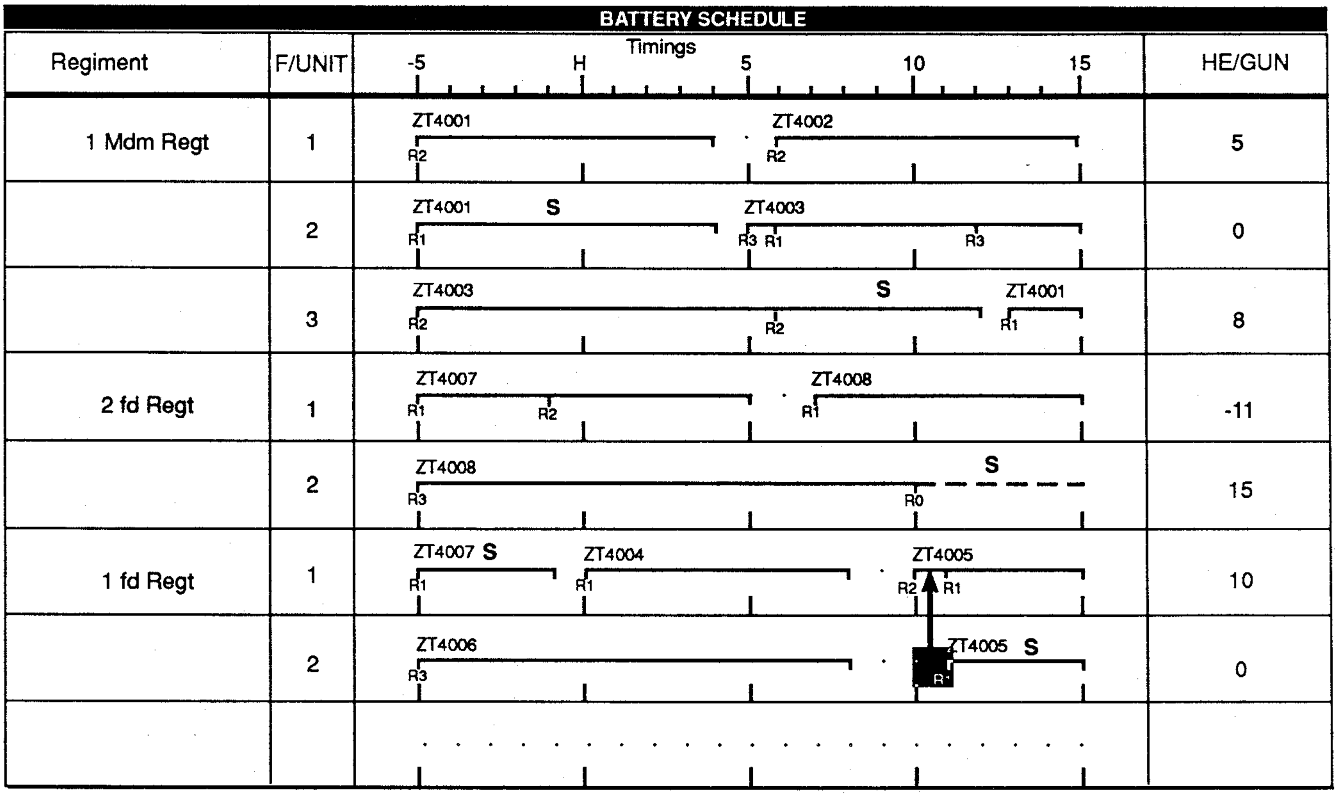

Figure I.9 Ammunition adjustment stage 2

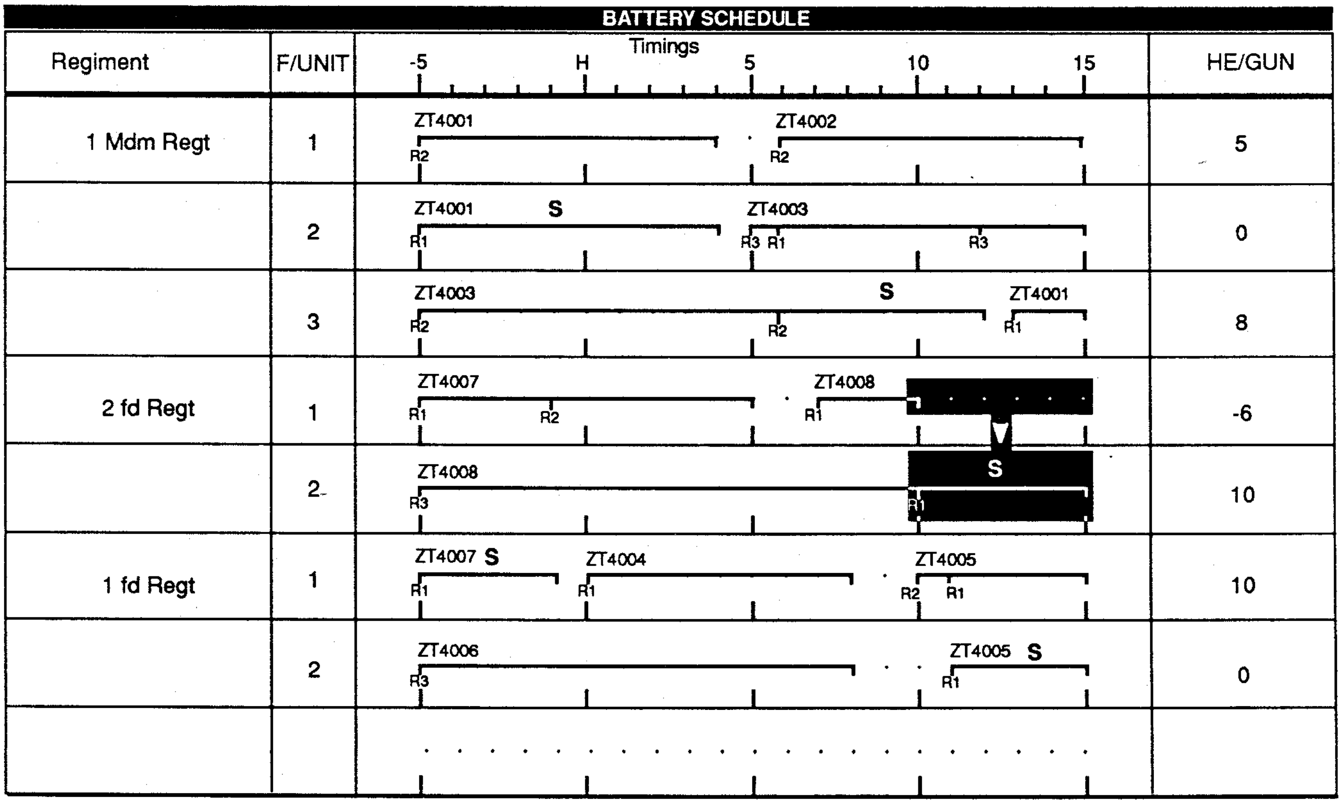

Figure I.10 Ammunition adjustment stage 3

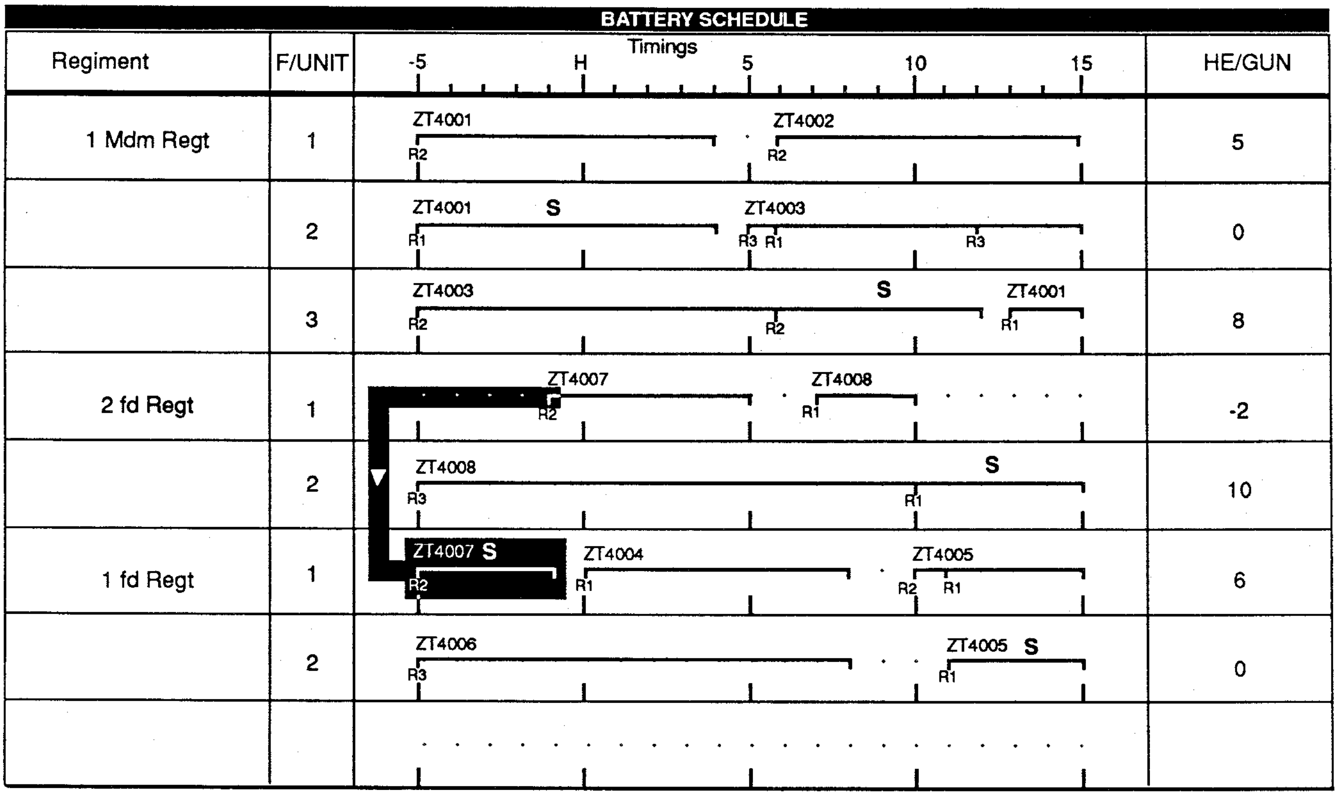

Figure I.11 Ammunition adjustment stage 4

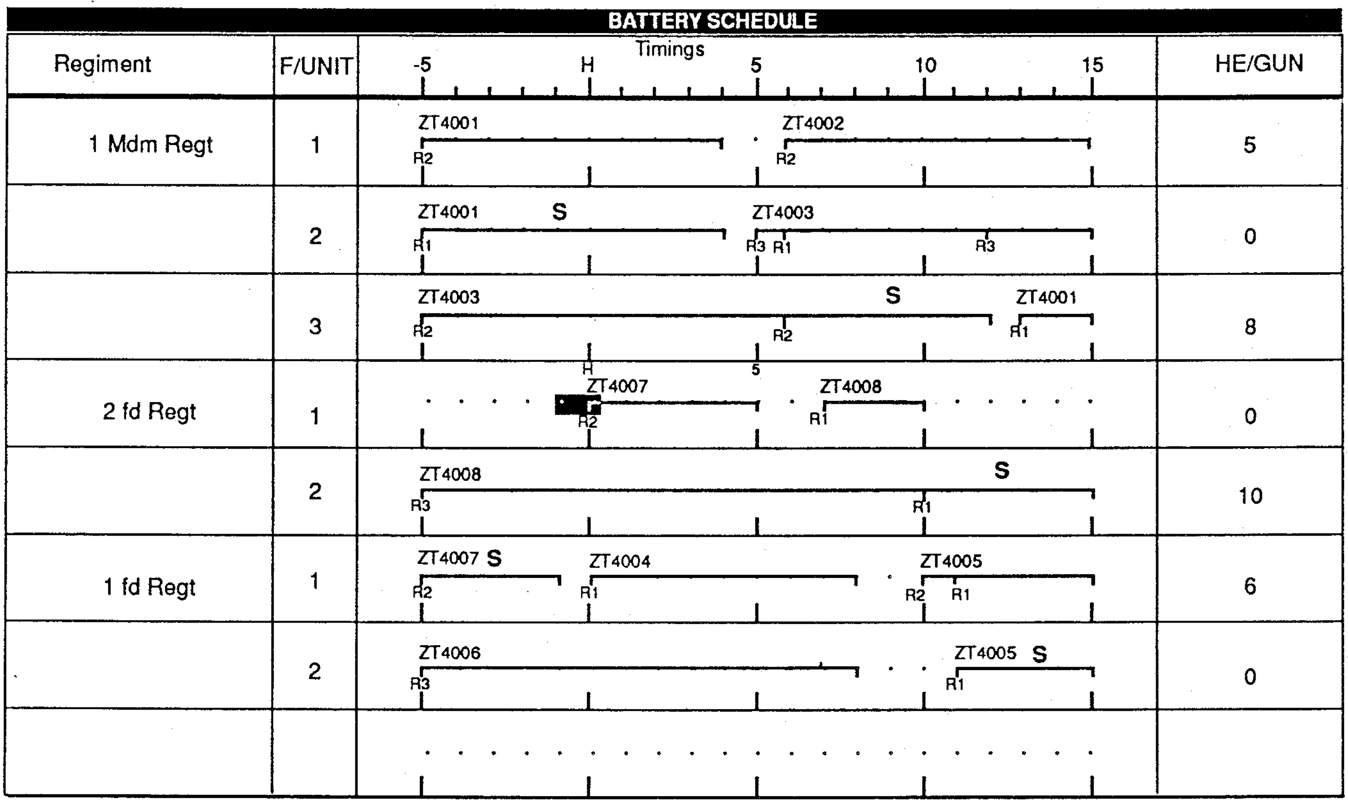

Figure I.12 Ammunition adjustment stage 5

DOCUMENT CONTROL DATA SHEET

Security classification of this page : UNCLASSIFIED

1 DOCUMENT NUMBERS

AR Number :	AR-006-721
Series Number :	WSRL-TR-25/91
Other Numbers :	

2 SECURITY CLASSIFICATION

a. Complete Document :	Unclassified
b. Title in Isolation :	Unclassified
c. Summary in Isolation :	Unclassified

3 DOWNGRADING / DELIMITING INSTRUCTIONS

4 TITLE

BATTERY-TARGET ALLOCATION SUBJECT TO FIRE PLANNING CONSTRAINTS

5 PERSONAL AUTHOR (S)

B. Das and
P.F. Calder

6 DOCUMENT DATE

March 1991

7

7. 1 TOTAL NUMBER OF PAGES	52
7. 2 NUMBER OF REFERENCES	3

8 8. 1 CORPORATE AUTHOR (S)

Weapons Systems Research Laboratory

8. 2 DOCUMENT SERIES and NUMBER
Technical Report
25/91

9 REFERENCE NUMBERS

a. Task :	ARM 86/174.2
b. Sponsoring Agency :	

10 COST CODE

11 IMPRINT (Publishing organisation)

Defence Science and Technology Organisation

12 COMPUTER PROGRAM (S)
(Title (s) and language (s))

13 RELEASE LIMITATIONS (of the document)

Approved for Public Release.

Security classification of this page : UNCLASSIFIED

Security classification of this page : UNCLASSIFIED

14 ANNOUNCEMENT LIMITATIONS (of the information on these pages)

No limitations

15 DESCRIPTORS

a. EJC Thesaurus Terms	Artillery fire Land combat Gun batteries Fire control systems
b. Non - Thesaurus Terms	

16 COSATI CODES

1506,
1905

17 SUMMARY OR ABSTRACT
(if this is security classified, the announcement of this report will be similarly classified)

(U)A system is described for allocating artillery batteries to targets for an artillery *Fire Plan*. An optimal allocation, which assigns batteries to targets in the most effective manner, is initially made. This is then modified to ensure that all batteries are firing over the whole duration of the fire plan while preserving the desired effect on each target. The resulting fire plan also satisfies ammunition constraints, ie, a battery is not scheduled to fire more ammunition than what it has. User interaction is supported to allow any specific requirements to be incorporated.

Security classification of this page : UNCLASSIFIED